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ELECTIVE 2 OPTIMAL CONTROL SYSTEMS (ACE 326)

Background Ref.1&2: Appendix A

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VECTOR & MATRIX ALGEBRA OUTLINES

- Matrices Definition
- Matrices Types
- Matrices Operations
- Solving n Linear Equations /n Unknowns

VECTOR & MATRIX ALGEBRA HISTORY

Linear algebra is the branch of mathematics concerning vector spaces/linear mapping between such spaces

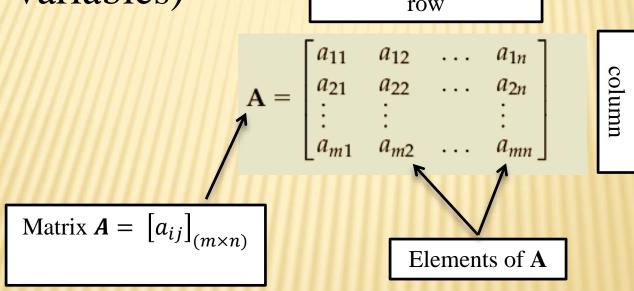
- $\Box \quad \text{Leibniz} (1693) \longrightarrow \text{Determinants}$
- □ Gabriel Cramer (1750-Cramer's rule) \longrightarrow solving linear systems

□ Gauss 1777–1855 \longrightarrow solving linear systems by Gaussian elimination method

Gottfried Wilhelm Leibniz (1646-1716): German mathematician and philosopher Gabriel Cramer (1704-1752): Swiss mathematician Johann Carl Friedrich Gauss (1777-1855): German mathematician

VECTOR & MATRIX ALGEBRA MATRICES DEFINITION

 A matrix is a rectangular array of quantities (real numbers/complex numbers/functions of variables)



VECTOR & MATRIX ALGEBRA MATRICES TYPES

Rectangle matrix

$$\boldsymbol{A} = \left[a_{ij}\right]_{(m \times n)}$$

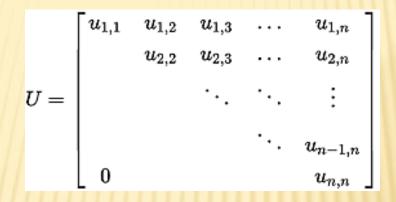
□ Square matrix

$$m = n \longrightarrow A = [a_{ij}]_{(n \times n)}$$

□ Vector m = 1 $\longrightarrow A = [a_{ij}]_{(1 \times n)} = \mathbf{a} \text{ (row vector)}$ n = 1 $\longrightarrow A = [a_{ij}]_{(m \times 1)} = \mathbf{a} \text{ (column vector)}$

VECTOR & MATRIX ALGEBRA MATRICES TYPES

Upper triangular matrix



Lower triangular matrix

$$L = egin{bmatrix} \ell_{1,1} & & 0 \ \ell_{2,1} & \ell_{2,2} & & \ \ell_{3,1} & \ell_{3,2} & \ddots & \ dots & dots & \ddots & \ddots & \ dots & dots & \ddots & \ddots & \ \ell_{n,1} & \ell_{n,2} & \dots & \ell_{n,n-1} & \ell_{n,n} \end{bmatrix}$$

Null Matrix

$$A = [a_{ij}]_{(m \times n)} = 0$$
$$AB = BA = 0$$

□ Unity Matrix (Identity Matrix) $A = I = I_n = [a_{ii}]_{(n \times n)} = 1; i = 1:n$

□ Singular Matrix/Nonsingular Matrix

$$|A| = 0$$

$$|A| \neq 0$$
Determinant of A-
Full rank

Symmetric Matrix

 $\boldsymbol{A} = \boldsymbol{A}^T$

Addition

$$C_{(m \times n)} = A + B; c_{ij} = a_{ij} + b_{ij}$$

Properties

1. Commutative $\longrightarrow A + B = B + A$ 2. Associative $\longrightarrow A + (B + C) = (A + B) + C$

3. $A + C = B + C \longrightarrow A = B$

4. $a_{ij} = b_{ij} \longrightarrow A_{(m \times n)} = B_{(m \times n)}$

Multiplication

$$C_{(m \times p)} = A_{(m \times n)} B_{(n \times p)}; c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

□ Properties

1. Commutative $\longrightarrow AB \neq BA$ 2. Associative $\longrightarrow A(BC) = (AB)C$

3. Distributive $\longrightarrow A(B + C) = AB + AC$ (A + B)C = AC + BC

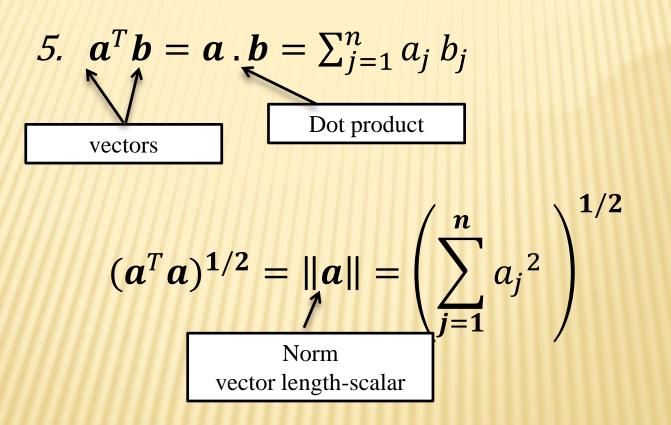
(A+B)(C+D) = AC + AD + BC + BD

□ Transpose of a matrix flips a matrix over its diagonal

$$A^{T} = A' \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Properties

1. $(A^T)^T = A$ 2. $(A + B)^T = A^T + B^T$ 3. $(\alpha A)^T = \alpha A^T; \alpha \text{ scalar}$ 4. $(AB)^T = B^T A^T$



Determinant is a scaling factor of a matrix

 $\det (A) = |A|$ $|A| = \sum_{j=1}^{n} a_{ij} \operatorname{cofac}(a_{ij}) \quad \operatorname{cofac}(a_{ij}) = (-1)^{i+j} |A_{ij}|$ for any *i* or *j*

Properties

1. $|A^T| = |A|$

□ Inverse of a matrix

$$(A)^{-1} = \frac{1}{|A|} adj(A)$$
$$adj(A) = (\operatorname{cofac}(a_{ij}))^T$$

Properties

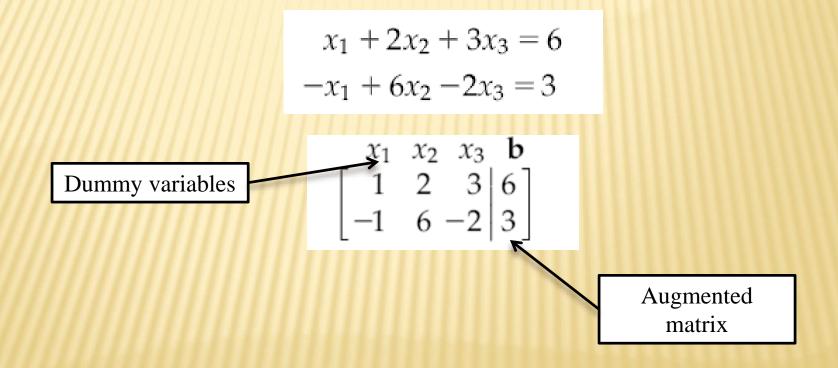
1. $(A^{-1})^{-1} = A$ 2. $(A^{-1})^{T} = (A^{T})^{-1}$ 3. $|A|^{-1} = |A^{-1}|$ 4. $(\alpha A)^{-1} = \alpha^{-1} A^{-1}; \alpha$ nonzero scalar

□ Elementary row/column operations

- 1. Interchange any two rows (columns).
- 2. Multiply any row (column) by a nonzero scalar.
- 3. Add to any row (column) a scalar multiple of another row (column).

VECTOR & MATRIX ALGEBRA SOLVING *n* LINEAR EQUATIONS/*n* UNKNOWNS

Linear equations are encountered in numerous engineering and scientific applications



VECTOR & MATRIX ALGEBRA SOLVING *n* LINEAR EQUATIONS/*n* UNKNOWNS

Let two linear equations:

$$a_{11}x_{1} + a_{12}x_{2} = b_{1}$$

$$a_{21}x_{1} + a_{22}x_{2} = b_{1}$$

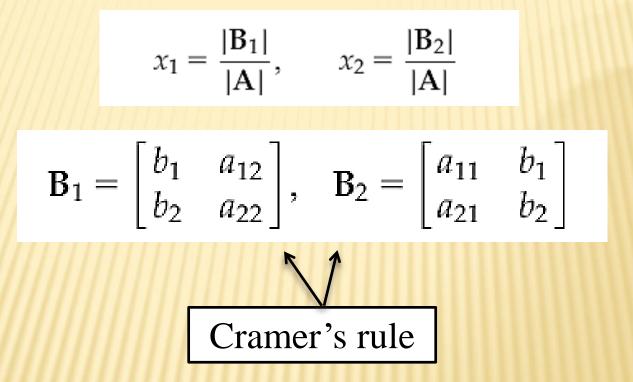
$$Ax = b$$
Elimination Process
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix} = \begin{bmatrix} b_{1} \\ b_{2} \end{bmatrix}$$

$$(a_{11}a_{22} - a_{12}a_{21})x_{1} = a_{22}b_{1} - a_{12}b_{2}$$

$$(a_{11}a_{22} - a_{12}a_{21})x_{2} = a_{11}b_{2} - a_{21}b_{1}$$

$$x_{1} = \frac{a_{22}b_{1} - a_{12}b_{2}}{a_{11}a_{22} - a_{12}a_{21}}, \quad x_{2} = \frac{a_{11}b_{2} - a_{21}b_{1}}{a_{11}a_{22} - a_{12}a_{21}}$$

VECTOR & MATRIX ALGEBRA SOLVING *n* LINEAR EQUATIONS/*n* UNKNOWNS



VECTOR & MATRIX ALGEBRA SUMMARY

\Box Solution of $n \times n$ system of linear equations

$$\geqslant |A| \neq 0 \quad \longrightarrow \quad$$

 $\geqslant |A| = 0 \longrightarrow$

Unique solution

Infinite number of solutions/No solution

VECTOR & MATRIX ALGEBRA SOLVING *m* LINEAR EQUATIONS/ *n* UNKNOWNS

Matrix rank is largest nonsingular square submatrix of given matrix $\longrightarrow r$

n > m, r ≤ m → full row rank matrix-∞ solutions
 m > n, r ≤ n → full column rank matrix-no solution/∞ solutions
 m = n, r = n → nonsingular matrix-unique solution

THANK YOU FOR YOUR ATTENTION