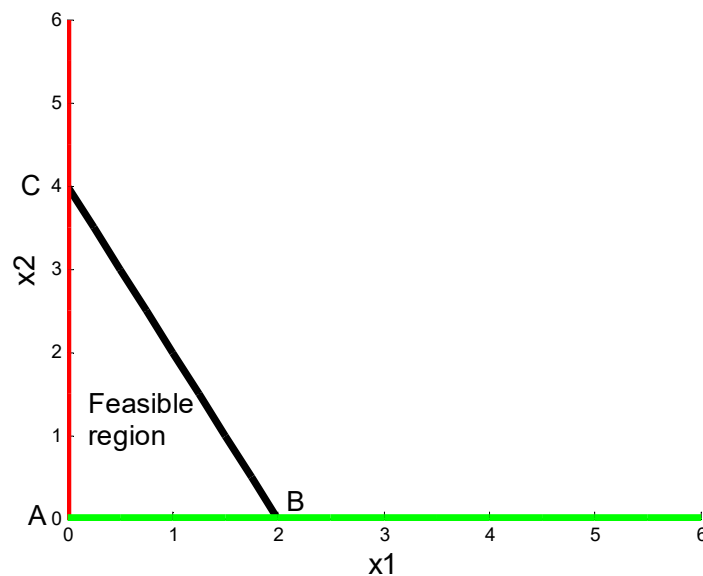




Exercise 1

1.1

$$\begin{aligned} &\text{Maximize } F(x_1, x_2) = x_1 + 2x_2 \\ &\text{subject to } 2x_1 + x_2 \leq 4 \\ &\quad x_1, x_2 \geq 0 \end{aligned}$$



$$A(0, 0), B(2, 0), \text{ and } C(0, 4)$$

$$f(0, 0) = 0 + 2(0) = 0$$

$$f(2, 0) = 2 + 2(0) = 2$$

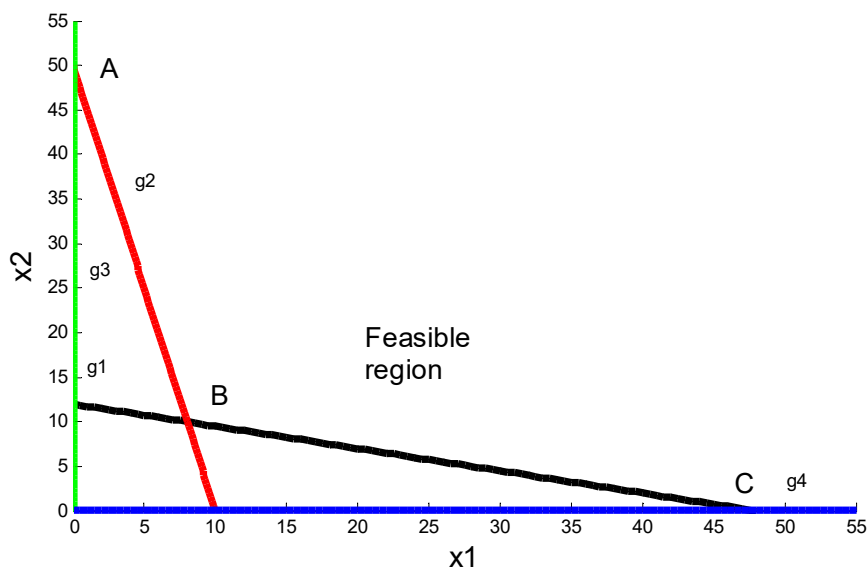
$$f(0, 4) = 0 + 2(4) = 8$$

The optimum point $x^* = (0, 4)$, $f^* = 8$



1.2

$$\begin{aligned} \text{Minimize } f(x_1, x_2) &= x_1 + 3x_2 \\ \text{subject to } x_1 + 4x_2 &\geq 48 \\ 5x_1 + x_2 &\geq 50 \\ x_1, x_2 &\geq 0 \end{aligned}$$



$$A(0, 50), B(8, 10), \text{ and } C(48, 0)$$

$$f(0, 50) = 150$$

$$f(8, 10) = 38$$

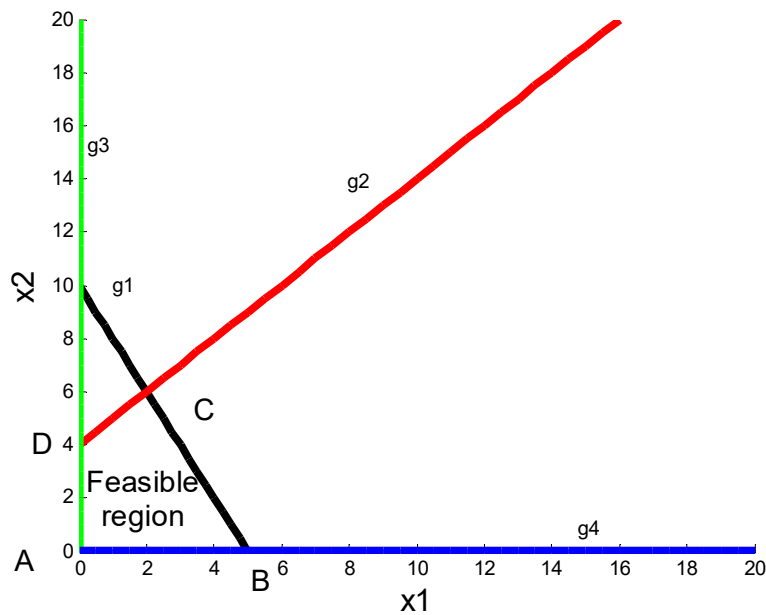
$$f(48, 0) = 48$$

The optimum point $x^* = (8, 10)$, $f^* = 38$



1.3

$$\begin{aligned} &\text{Minimize } f(x_1, x_2) = 5x_1 + 10x_2 \\ &\text{subject to } 10x_1 + 5x_2 \leq 50 \\ &\quad \quad \quad 5x_1 - 5x_2 \geq -20 \\ &\quad \quad \quad x_1, x_2 \geq 0 \end{aligned}$$



$$A(0, 0), B(5, 0), C(2, 6), \text{ and } D(0, 4)$$

$$f(0, 0) = 0$$

$$f(5, 0) = 25$$

$$f(2, 6) = 70$$

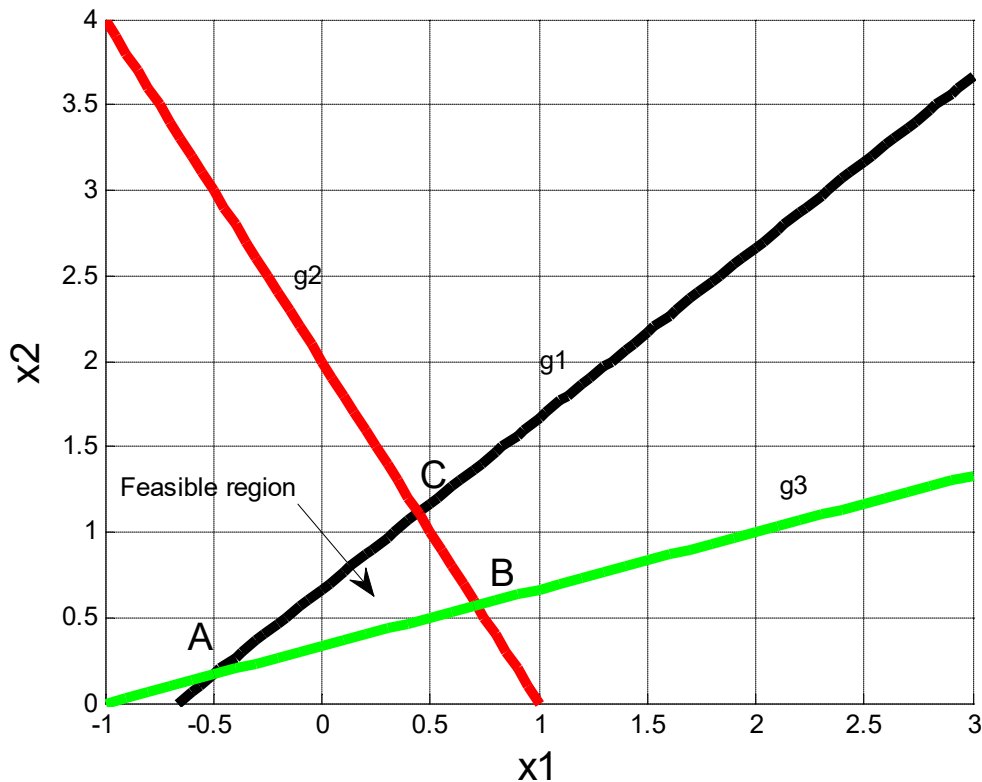
$$f(0, 4) = 40$$

The optimum point $x^* = (0, 0)$, $f^* = 0$



1.5

$$\begin{aligned} &\text{Minimize } f(x_1, x_2) = 3x_1 + 6x_2 \\ &\text{subject to } -3x_1 + 3x_2 \leq 2 \\ &\quad 4x_1 + 2x_2 \leq 4 \\ &\quad -x_1 + 3x_2 \geq 1 \end{aligned}$$



$$A(-0.5, 0.2), B(0.7, 0.6), \text{ and } C(0.5, 1.1)$$

$$f(-0.5, 0.2) = -0.3$$

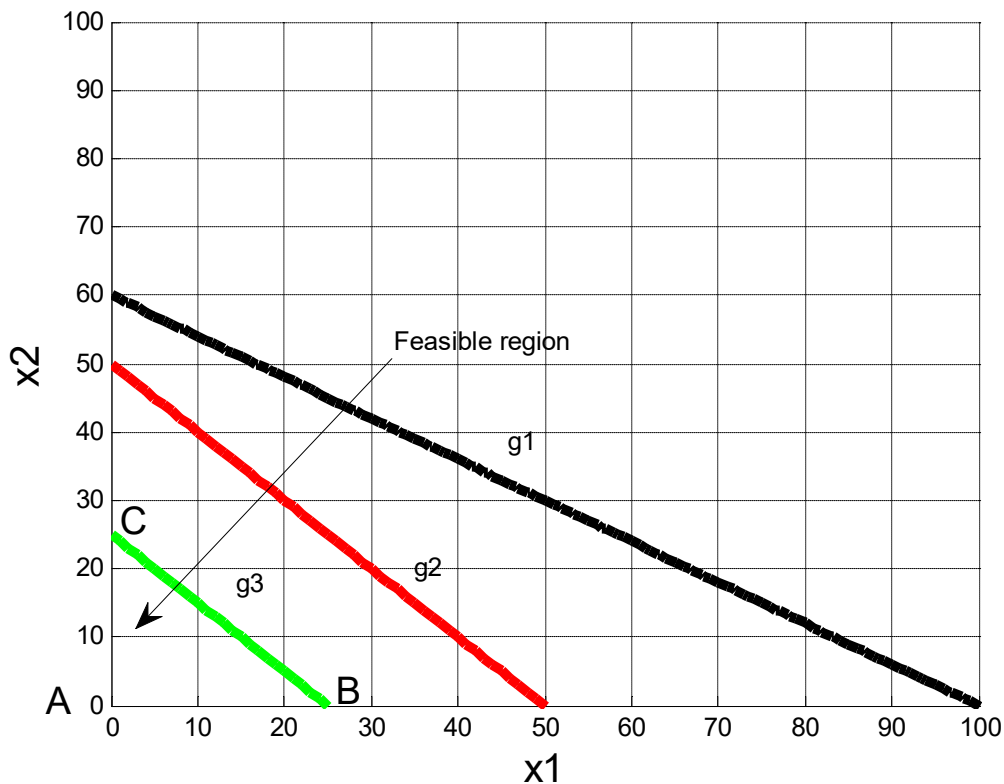
$$f(0.7, 0.6) = 5.7$$

$$f(0.5, 1.1) = 8.1$$

The optimum point $x^* = (-0.5, 0.2)$, $f^* = -0.3$



1.8 A trucking company wants to purchase two new trucks. It has \$3 million to spend. The prices of the two kinds of trucks are \$30 thousands and \$50 thousands. There are some limitations on the operations that need to be considered. The company can hire at most 50 truck drivers. Garage and maintenance facilities can handle at the most 25 trucks. How many trucks of each type should the company purchase?



$$30000x_1 + 50000x_2 \leq 3000000$$

$$x_1 + x_2 \leq 50$$

$$x_1 + x_2 \leq 25$$

The optimum point is any point on line CB without points C & B