



Exercise 2

2.1 Find the local minimum points using necessary/sufficient conditions:

- $f(x) = 3x^2 + 2x + 7$
- $f(x) = x^3 + 2x^2 - 3x - 6$
- $f(x) = 20/x + 5x$
- $f(x) = \cos x$
- $f(x) = x^2 e^{-x}$

2.2 Find the local minimum points using necessary/sufficient conditions:

- $f(x_1, x_2) = 3x_1^2 + 2x_1x_2 + 2x_2^2 + 7$
- $f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2$
- $f(x_1, x_2) = 7x_1^2 - x_1 + 12x_2^2$
- $f(x_1, x_2) = 25x_1^2 + 20x_2^2 - 2x_1 - x_2$

2.3 Find the candidate local minimum points using necessary conditions:

- Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$,
Subject to $x_1 + x_2 = 4$
- Minimize $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 + 1)^2$,
Subject to $2x_1 + 3x_2 - 4 = 0$
- Maximize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8$,
Subject to $x_1 + x_2 = 4$
- Minimize $f(x_1, x_2) = 4x_1^2 + 9x_2^2 + 6x_2 - 4x_1 + 13$,
Subject to $x_1 - 3x_2 + 3 = 0$

2.4 Find the candidate local minimum points using KKT necessary conditions:

- Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8$,
Subject to $x_1 + x_2 \leq 4$
- Maximize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8$,
Subject to $x_1 + x_2 \leq 4$
- Minimize $f(x_1, x_2) = x_1^3 - 16x_1 - 3x_2^2 + 2x_2$,
Subject to $x_1 + x_2 \leq 3$
- Minimize $f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 2x_2 + 6$,
Subject to $x_1 + x_2 \geq 4$



- e. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$,
Subject to $x_1 + x_2 \geq 4$ and $x_1 - x_2 = 2$
- f. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$,
Subject to $x_1 + x_2 \geq 4$ and $x_1 - x_2 \geq 2$

2.5 Consider the following problem with equality constraints:

$$\text{Minimize } f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$$

$$\text{Subject to } x_1 + x_2 - 4 = 0 \text{ and } x_1 - x_2 - 2 = 0$$

- a. Is it a valid optimization problem? Explain.
- b. Explain how you would solve the problem? Are necessary conditions needed to find the optimum solution?