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جامعة المنوفية، كلية الهندسة الالكترونية

قسم هندسة الالكترونيات الصناعية والتحكم

Exercise 2

2.1 Find the local minimum points using necessary/sufficient conditions:

a. $f(x) = 3x^2 + 2x + 7$,

$[\boldsymbol{x^* = -1/3}]$

b. $f(x) = x^3 + 2x^2 - 3x - 6$,

$[\boldsymbol{x^* = 0.5352}]$

c. $f(x) = 20/x + 5x$,

$[\boldsymbol{x^* = 2}]$

d. $f(x) = cosx$,

$[\boldsymbol{x^* = (2n+1)\pi; n = 0, \pm 1, \pm 2, \dots}]$

e. $f(x) = x^2 e^{-x}$,

$[\boldsymbol{x^* = 0}]$

2.2 Find the local minimum points using necessary/sufficient conditions:

a. $f(x_1, x_2) = 3{x_1}^2 + 2x_1x_2 + 2{x_2}^2 + 7$,

$[\boldsymbol{x^* = (0\ ,0)}]$

b. $f(x_1, x_2) = {x_1}^2 + x_1x_2 + {x_2}^2$,

$[\boldsymbol{x^* = (0\ ,0)}]$

c. $f(x_1, x_2) = 7{x_1}^2 - x_1 + 12{x_2}^2$,

$[\boldsymbol{x^* = (\frac{1}{14}, 0)}]$

d. $f(x_1, x_2) = 25{x_1}^2 + 20{x_2}^2 - 2x_1 - x_2$,

$[\boldsymbol{x^* = (\frac{1}{25}, \frac{1}{40})}]$

2.3 Find the candidate local minimum points using necessary conditions:

a. Minimize $f(x_1, x_2) = 4{x_1}^2 + 3{x_2}^2 - 5x_1x_2 - 8x_1$,
Subject to $x_1 + x_2 = 4$,

$[\boldsymbol{x^* = \left(\frac{13}{16}, \frac{11}{16}\right), v^* = -\frac{1}{6}}]$

b. Minimize $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 + 1)^2$,
Subject to $2x_1 + 3x_2 - 4 = 0$,

$[\boldsymbol{x^* = \left(\frac{32}{13}, \frac{-4}{13}\right), v^* = -\frac{6}{13}}]$

c. Maximize $f(x_1, x_2) = 4{x_1}^2 + 3{x_2}^2 - 5x_1x_2 - 8$,
Subject to $x_1 + x_2 = 4$,

$$\left[x^* = \left(\frac{11}{6}, \frac{13}{6}\right), v^* = \frac{23}{6}\right]$$

d. Minimize $f(x_1, x_2) = 4{x_1}^2 + 9{x_2}^2 + 6x_2 - 4x_1 + 13$,
Subject to $x_1 - 3x_2 + 3 = 0$,

$$[x^* = (-0.4, 0.8667), v^* = 7.2]$$

2.4 Find the candidate local minimum points using KKT necessary conditions:

a. Minimize $f(x_1, x_2) = 4{x_1}^2 + 3{x_2}^2 - 5x_1x_2 - 8$,
Subject to $x_1 + x_2 \leq 4$,

$$[x^* = (0, 0)]$$

b. Maximize $f(x_1, x_2) = 4{x_1}^2 + 3{x_2}^2 - 5x_1x_2 - 8$,
Subject to $x_1 + x_2 \leq 4$,

$$\left[x^* = \left(\frac{11}{6}, \frac{13}{6}\right), x^* = (0, 0)\right]$$

c. Minimize $f(x_1, x_2) = {x_1}^3 - 16x_1 - 3{x_2}^2 + 2x_2$,
Subject to $x_1 + x_2 \leq 3$,

$$[x^* = (\pm 4/\sqrt{3}, 1/3), x^* = (0, 3), x^* = (2, 1)]$$

d. Minimize $f(x_1, x_2) = {x_1}^2 + {x_2}^2 - 4x_1 - 2x_2 + 6$,
Subject to $x_1 + x_2 \geq 4$,

$$\left[x^* = \left(\frac{5}{2}, \frac{3}{2}\right), u^* = 1\right]$$

e. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$,
Subject to $x_1 + x_2 \geq 4$ and $x_1 - x_2 = 2$,

$$[x^* = (3, 1), u^* = 2, v^* = -2]$$

f. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$,
Subject to $x_1 + x_2 \geq 4$ and $x_1 - x_2 \geq 2$,

$$[x^* = (3, 1), {u_1}^* = 2, {u_2}^* = 2]$$

2.5 Consider the following problem with equality constraints:

Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$

Subject to $x_1 + x_2 - 4 = 0$ and $x_1 - x_2 - 2 = 0$

a. Is it a valid optimization problem? Explain.
b. Explain how you would solve the problem? Are necessary conditions needed to find the optimum solution?