Elective 2
Optimal Control Systems
(ACE 326)

Homework 1

Lecture 3

Exercise 1

$$f(\boldsymbol{x}) = -x_1 - 0.5x_2$$

Subject to $2x_1+3\,x_2 \leq 12, 2x_1+x_2 \leq 8, -x_1, -x_2 \leq 0$

Solution:

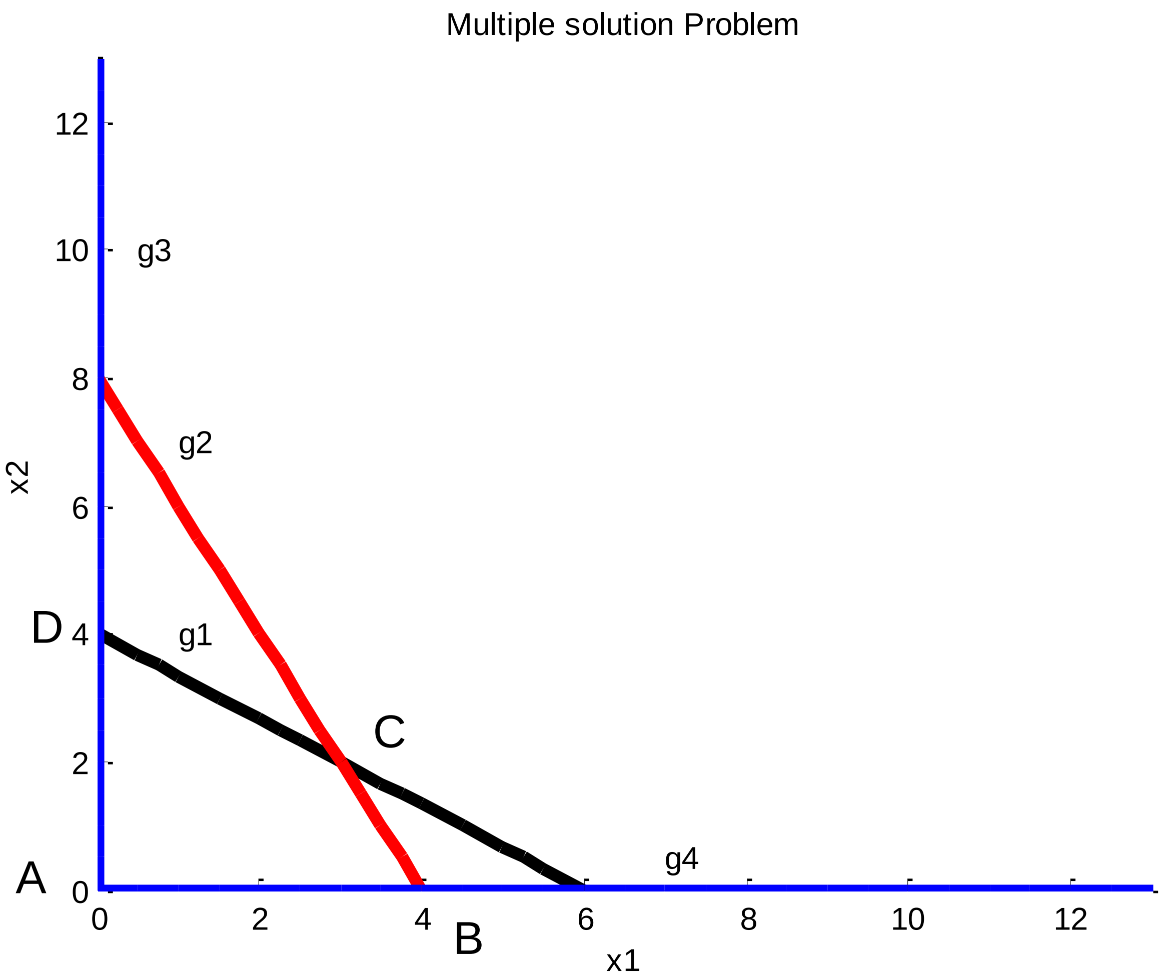
1. **Write the equation of any inequality in standard form:**

a. $2x_1 + 3x_2 = 12\,(g1)$

b. $2x_1 + x_2 = 8 \quad (g_2)$

c. $-x_1, -x_2 \leq 0$ 1^{st} quadrant $(g_3 and g_4)$

Exercise 1



Multiple solution Problem
12
10
8
6
4
2
0
x2
g3
g2
g1
D
C
A
g4
0
2
4
6
8
10
12
B
x1

Exercise 1

2. **Determine the vertices of the feasible region:**

$A(0,0)$, $B(4,0)$, $C(3,2)$, **and** $D(0,4)$

Determine the objective function at each vertex:

$$f(\mathbf{0},\mathbf{0}) = 0$$
$$\boldsymbol{f(4,0) = -4}$$
$$\boldsymbol{f(3,2) = -4}$$
$$f(\mathbf{0},\mathbf{4}) = -2$$

Multiple Solutions

Exercise 2

$$f(\boldsymbol{x}) = -x_1 + 2x_2$$

Subject to $-2x_1 + x_2 \leq 0, -2x_1 + 3x_2 \leq 6, -x_1, -x_2 \leq 0$

Solution:

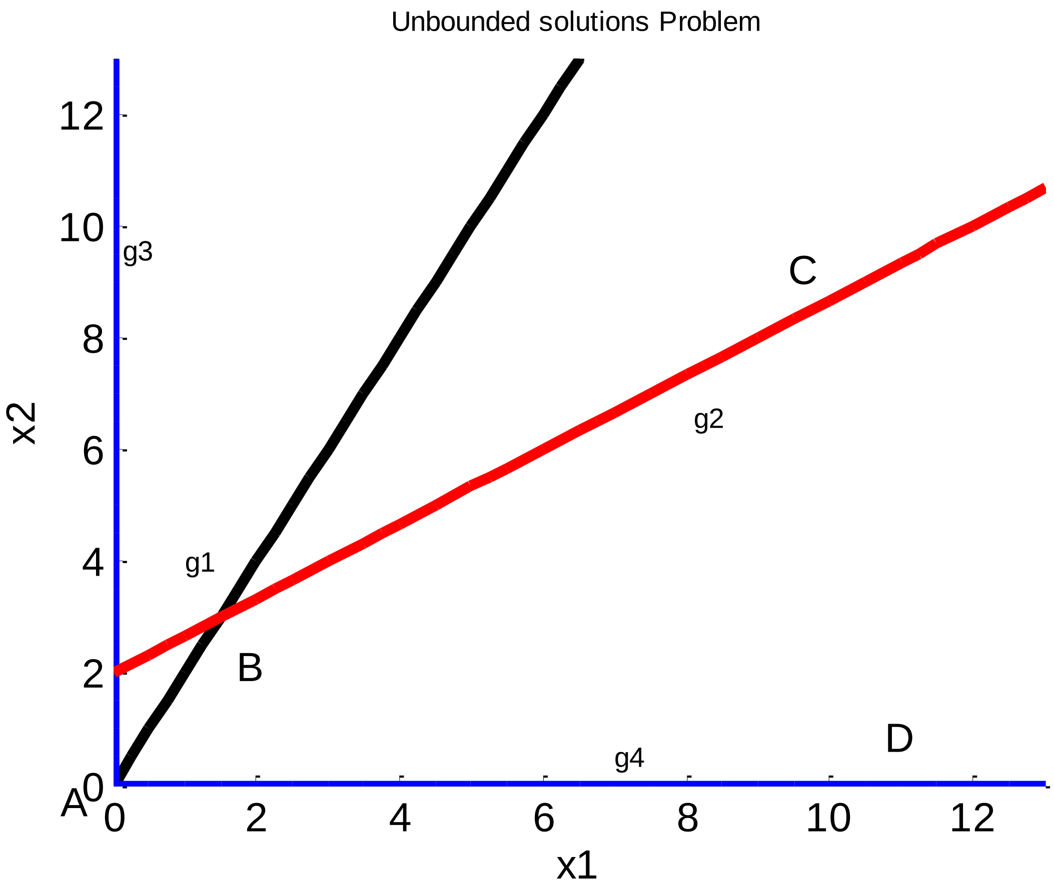
1. Write the equation of any inequality in standard form:

a. $-2x_1 + x_2 = 0$ $(g1)$

b. $-2x_1 + 3x_2 = 6$ (g_2)

c. $-x_1, -x_2 \leq 0$ 1st quadrant $(g_3 and g_4)$

Exercise 2


Unbounded solutions Problem
12
10
8
6
4
2
0
0
2
4
6
8
10
12
x2
x1
g1
g2
g3
g4
A
B
C
D

Exercise 2

2. **Determine the vertices of the feasible region:**

$A(0,0)$, $B(1.5,3)$, C, and D

Determine the objective function at each vertex:

$$f(\mathbf{0},\mathbf{0}) = 0$$

$$f(1.5,3) = 4.5$$

$$f_C \text{ and } f_D$$ ← Unbounded Solutions

Exercise 3

$$f(\boldsymbol{x}) = x_1 + 2x_2$$

Subject to $3x_1 + 2x_2 \leq 6, 2x_1 + 3x_2 \geq 12$

$$x_1, x_2 \leq 5$$

Solution:

$$x_1, x_2 \geq 0$$

1. Write the equation of any inequality in standard form:

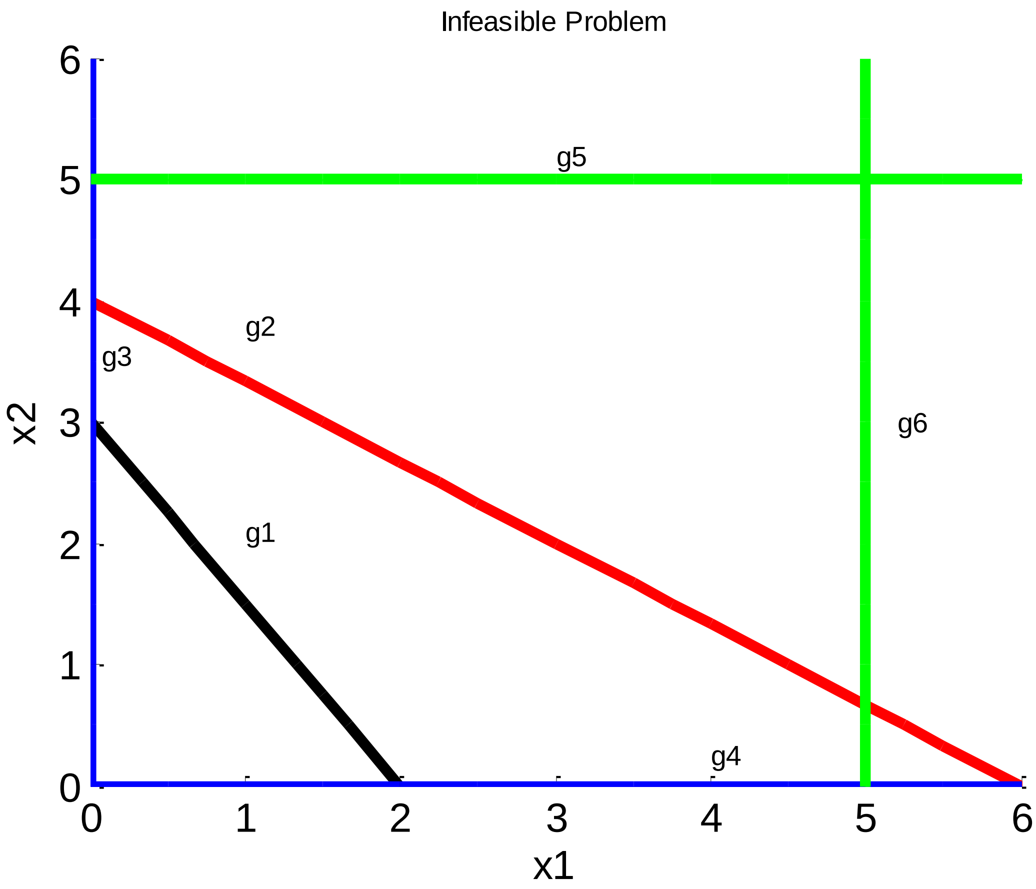
a. $3x_1 + 2x_2 = 6$ $(g1)$

b. $2x_1 + 3x_2 = 12$ (g_2)

c. $x_1, x_2 \geq 0$ 1st quadrant $(g_3 and g_4)$

d. $x_1, x_2 \leq 5$

Exercise 3



Remarks

- The optimization problem that can be solved graphically should be linear, continuous/discrete, and static with **two design variables** .
- The maximization problem equals the maximum cost function of all contour vertices.
- The minimization problem equals the minimum cost function of all contour vertices.