ELECTIVE 2
OPTIMAL CONTROL SYSTEMS
(ACE 326)

Lecture 1- Introduction to Optimization Theory

Ref. 1: Chapters 1&2

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INTRODUCTION TO OPTIMIZATION THEORY
OUTLINES

- **What Is Optimization?**
- **Why Is Optimization?**
- **How Is Optimization?**
- **Optimum Design Problem Formulation**

INTRODUCTION TO OPTIMIZATION THEORY
WHAT IS OPTIMIZATION?

- **Optimization=Best Design**
- **A branch of mathematics encompasses minimization and/or maximization**
  - Pythagoras (569 BC to 475 BC) defined optimum over numbers, geometrical shapes, physics, astronomy
  - Bellman (1957) introduced optimality principle for dynamic programing problem
  - Russell C. Eberhart and James Kennedy (1995) Particle Swarm Optimization (PSO)

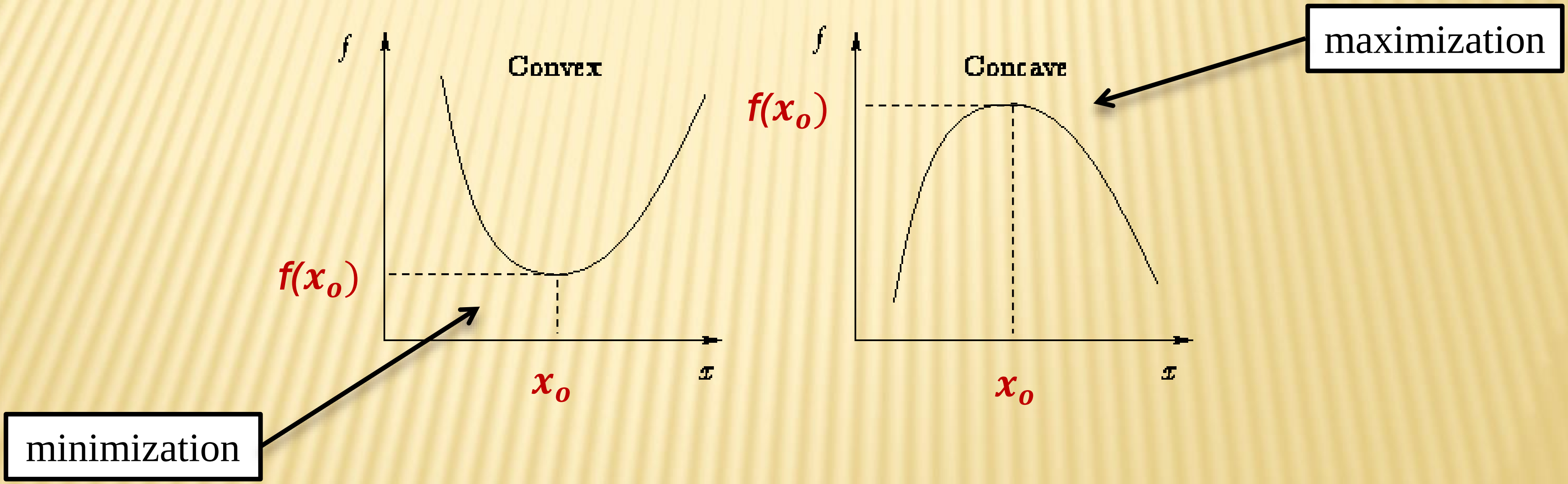
Pythagoras of Samos (569 BC to 475 BC): a Greek philosopher and mathematician
Richard Ernest Bellman (1920 –1984): an American mathematician
Russell C. Eberhart: an American electrical engineer
James Kennedy : an American social psychologist

INTRODUCTION TO OPTIMIZATION THEORY
WHAT IS OPTIMIZATION?

- **Mathematically:**
- *Given:* a function $f \in R, there\ is\ an\ element$

$$x_o \mid \; f(x_o) \leq f(x)\ \forall\ x \text{ "minmization" } or$$

$$f(x_o) \geq f(x)\ \forall\ x \text{ "}maxmization\text{"}$$



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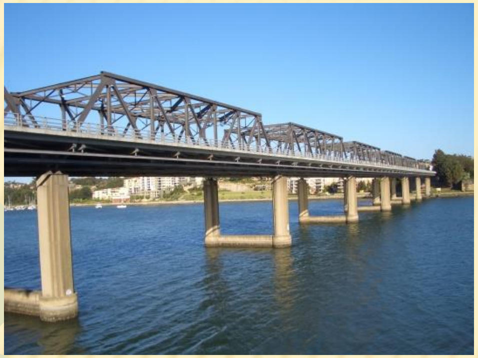
WHY IS OPTIMIZATION?

Civil Engineer



Mechanical Engineer

Chemical Engineer

Electronic Engineer

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WHY IS OPTIMIZATION?

Competitive marketplace

Maximize the load a robot can lift

Design the smallest heat exchanger that accomplishes the desired heat transfer

Design the lowest-cost bridge for the site

Optimization

Minimization

- ✓ Cost
- ✓ Energy

Maximization

- ✓ Strength
- ✓ Reliability
- ✓ Longevity
- ✓ Efficiency
- ✓ Utilization

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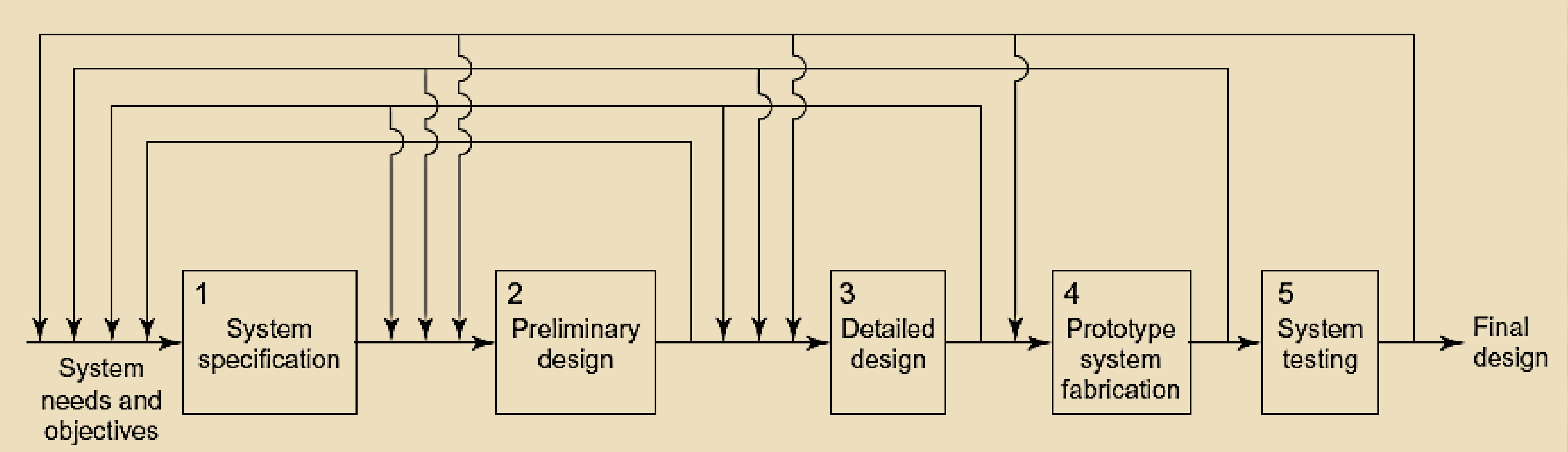
HOW IS OPTIMIZATION?



System Evolution Model

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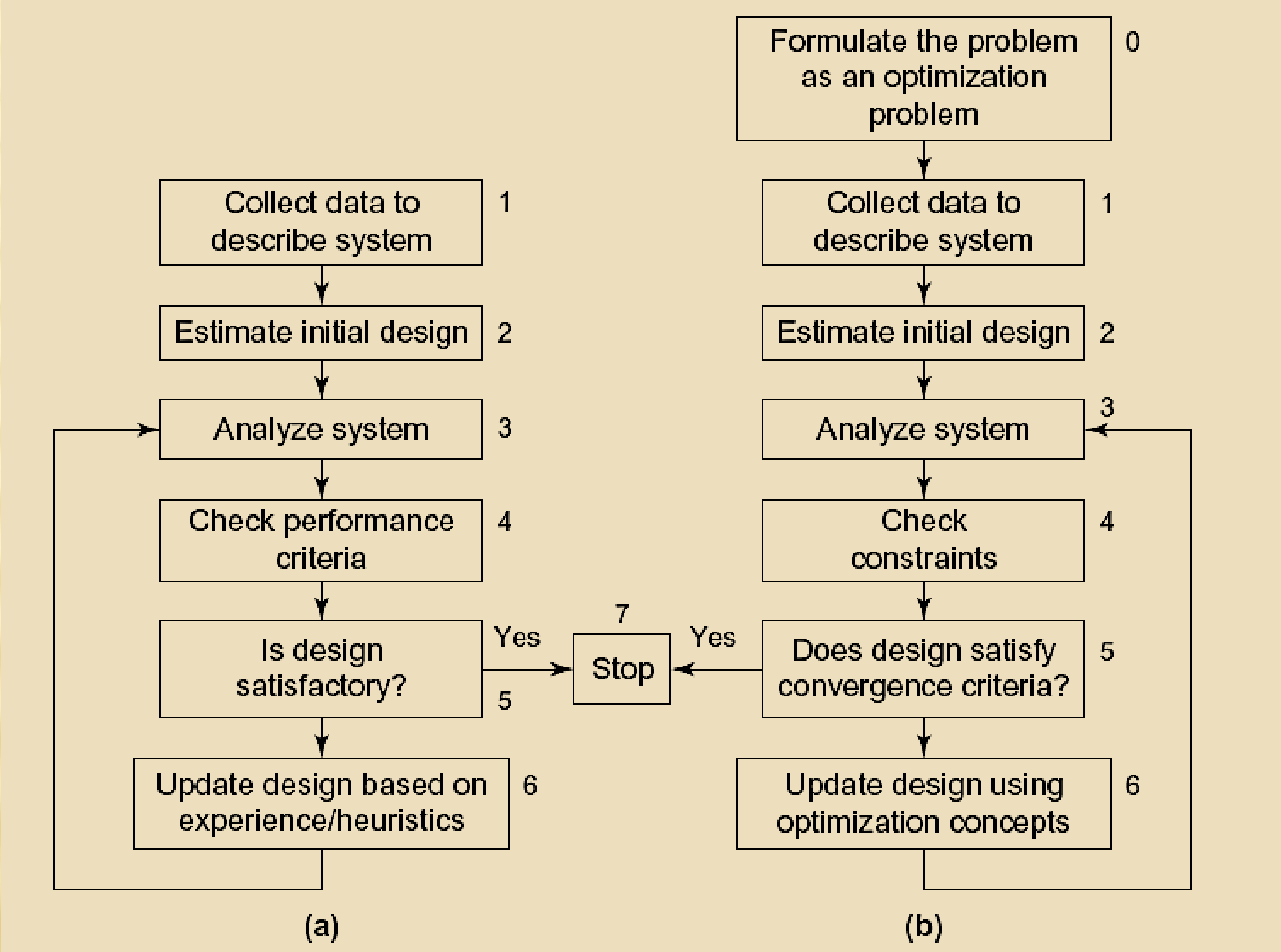
HOW IS OPTIMIZATION?



(a) Conventional Design Method

(a) Optimum Design Method

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HOW IS OPTIMIZATION?

Optimum Design Steps
Step1: Project/Problem Description
Step2: Data & Information Collection
Step3: Definition of Design Variables
Step4: Optimization Criteria
Step5: Formulation of Constraints

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HOW IS OPTIMIZATION?

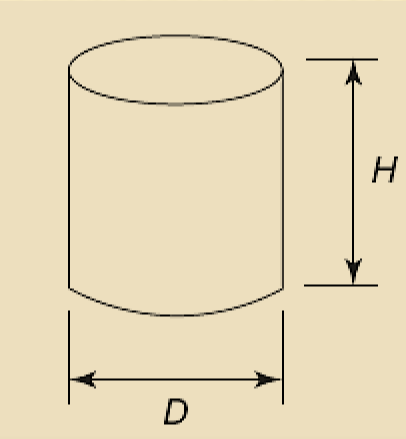
- **Example 1: Design a can to hold at least 400 ml of liquid with the following requirements:**
  - Minimize its manufacturing cost
  - Diameter should be no more than 8 cm and no less than 3.5 cm
  - Height should be no more than 18 cm and no less than 8 cm

- **P.S. Cost is directly related to the surface area of the sheet metal used**

HOW IS OPTIMIZATION?

Optimum Design Steps

Step1: Project/Problem Description



Step2: Data & Information Collection

✓ Data are given in the problem statement

Step3: Definition of Design Variables

- ✓ $D =$ diameter of the can, cm
- ✓ $H =$ height of the can, cm

Step4: Optimization Criteria

- ✓ The design objective is to minimize the cost; minimize the total surface area S

$$S = \pi DH + 2\left(\frac{\pi}{4}D^2\right), cm^2$$

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HOW IS OPTIMIZATION?

Step5: Formulation of Constraints

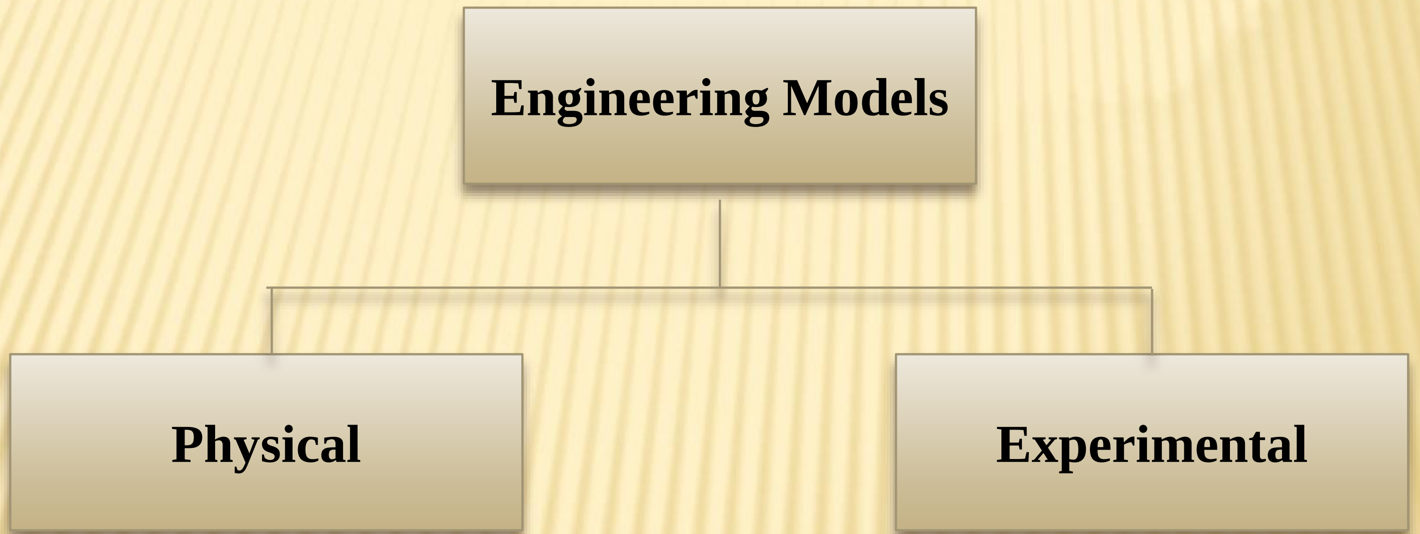
$$\left(\frac{\pi}{4}D^2\right)H \geq 400, cm^3$$

$$3.5 \leq D \leq 8, cm$$

$$8 \leq H \leq 18, cm$$

ENGINEERING MODELS IN OPTIMIZATION

- **Proper Problem definition & formulation = 50 % of total effort needed to solve it**



- ✓ Design variables
- ✓ Material properties

- ✓ Speed
- ✓ Heat transfer
- ✓ Cost
- ✓ Efficiency

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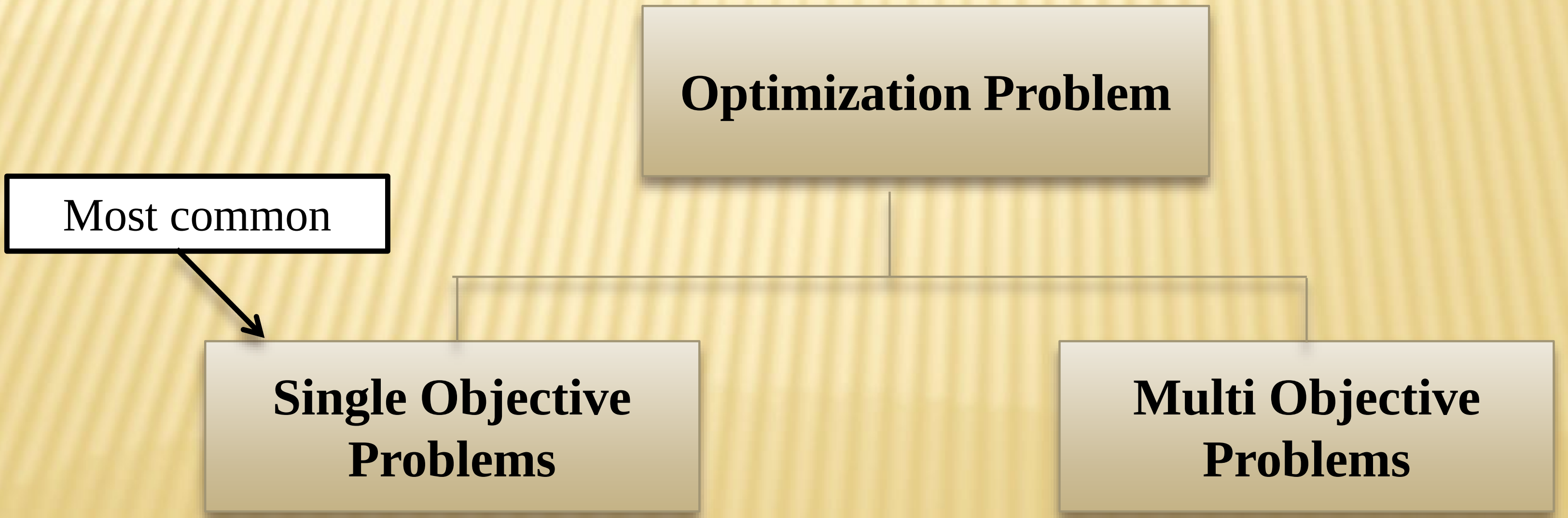
ENGINEERING MODELS IN OPTIMIZATION---CONT.

- **Design variables should be independent** ⟶

  Degrees of freedom ⟶ **Design space**
- **Good design optimization model=90% optimum design**
- **Optimization Criteria=Objective function=Cost function=Performance index**



- ✓ Minimize Cost or Weight
- ✓ Minimize Cost& Weight

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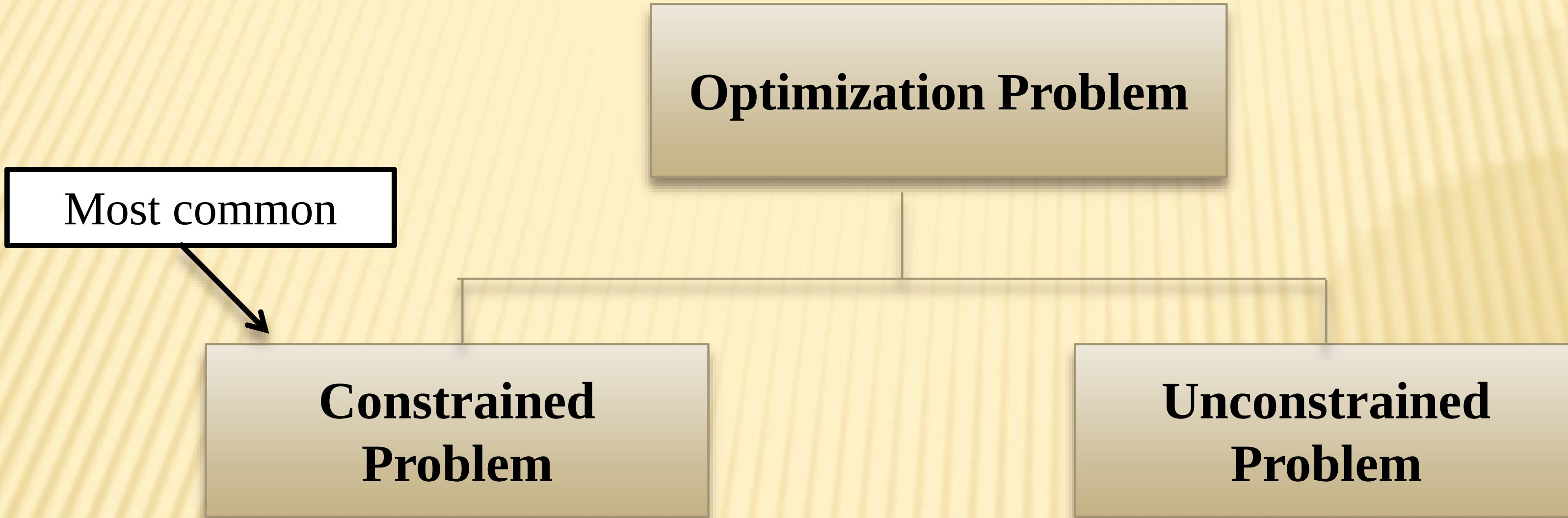
CONSTRAINED/UNCONSTRAINED PROBLEMS



- ✓ Equality ⟶ $r = 3$
- ✓ Inequality ⟶ $r \geq 3 \; or \; h \leq 10$
- ✓ Two allowable values ⟶ $3 \leq s \leq 7$
- ✓ Function ⟶ $Actual\ speed \leq Desired\ speed$

  $Actual\ speed - Desired\ speed \leq 0$
- ✓ Parametric ⟶ $S(\emptyset) < 100, \quad 0 \leq \emptyset \leq 2\pi$

OPTIMIZATION PROBLEMS TYPES

Optimization Problem

Continuous/Discrete

- ✓ Design variables
- ✓ Constraints

- $x_1 \in R$ **(Continuous variable)**
- $x_1 = \{10,20,50\}$ **(Discrete variable)**

Linear/Nonlinear

- ✓ Function
- ✓ Constraints

- $\boldsymbol{f}(\boldsymbol{x}) = x_1 + x_2$
- $\boldsymbol{f}(\boldsymbol{x}) = {x_1}^2 \, x_2$
- $x_1 x_2 \geq 20$

Smooth/Non-smooth

- ✓ Function

- Differentiable
- Non- differentiable

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OPTIMIZATION PROBLEMS TYPES---CONT.

Optimization Problem

- **……..**
- **Static/Dynamic**
  - ✓ Design variables
  - ✓ Constraints
  - $S = \pi DH + 2\left(\frac{\pi}{4}D^2\right)$
  - $f\left(\boldsymbol{x}(t)\right) = x_1(t) + x_2(t)$
  - $x_1(t) \leq 10$

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STANDARD DESIGN OPTIMIZATION MODEL

- Find a vector $\boldsymbol{x}$ =[x_1, x_2 ⋯, x_n] of design variables to minimize a cost function:

$$f(\boldsymbol{x}) = f(x_i);\ \ i = 1{:}\,n$$

subject to:

- m equality constraints:

$$h_j(\boldsymbol{x}) = h_j(x_1, x_2 \cdots, x_n) = 0;\ \ j = 1{:}\,m$$

- p inequality constraints:

$$g_l(\boldsymbol{x}) = g_l(x_1, x_2 \cdots, x_n) \le 0;\ \ l = 1{:}\,p$$

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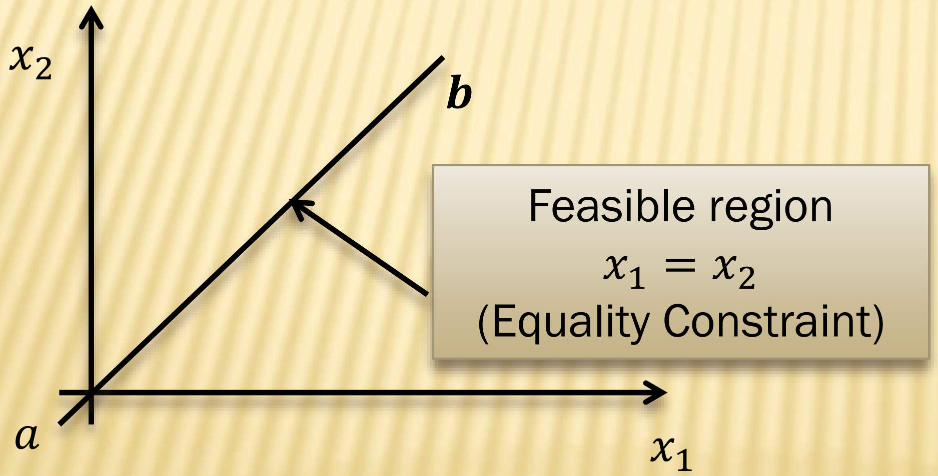
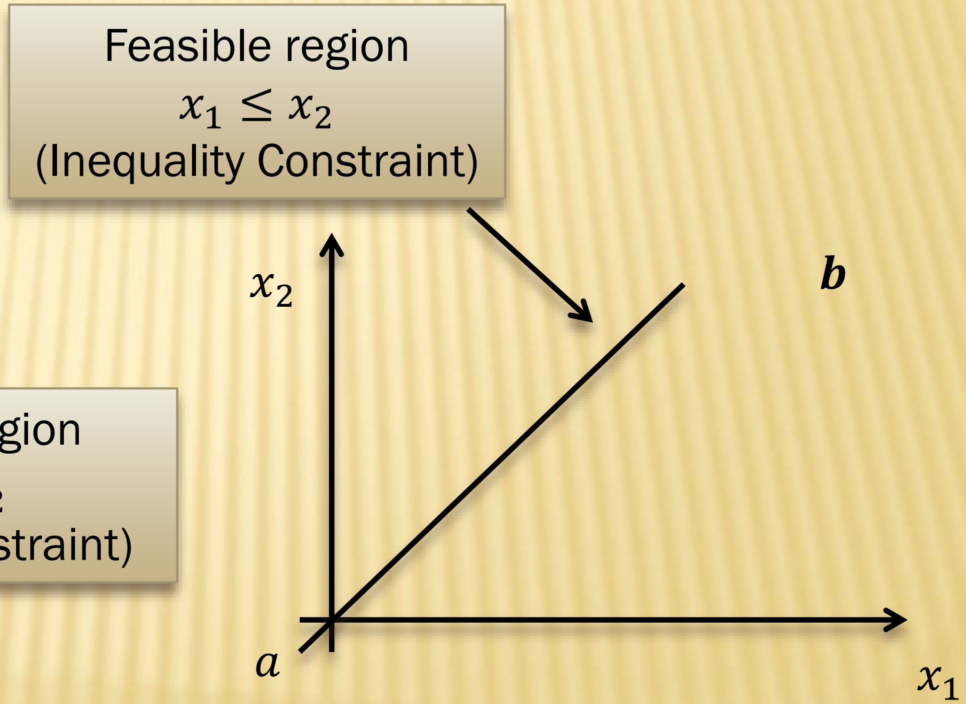
IMPORTANT OBSERVATIONS

- **Feasible design (acceptable): design meets all requirements**
- **Infeasible design (unacceptable):design does not meet all requirements**





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IMPORTANT OBSERVATIONS

- **More Constraints** ⟶ **Shrink feasible region** ⟶ **Reduce number of possible designs**

- **Let *S* is collection of design points satisfy all constraints**

$$S = \{\boldsymbol{x} \mid h_j(\boldsymbol{x}) = 0, j = 1{:}m; g_l(\boldsymbol{x}) \leq 0;\ \ l = 1{:}p\}$$

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IMPORTANT OBSERVATIONS

- Functions $f(\boldsymbol{x})$, $h_j(\boldsymbol{x})$, and $g_l(\boldsymbol{x})$ must depend, explicitly or implicitly, on some of design variables.
- Functions do not depend on any variable have can be safely ignored
- Number of equality constraints must be less than, or at most equal to number of design variables

$$m \leq n$$

$$m > n$$

(inconsistent-no solution)

$$m = n$$

(no optimization problem is necessary)

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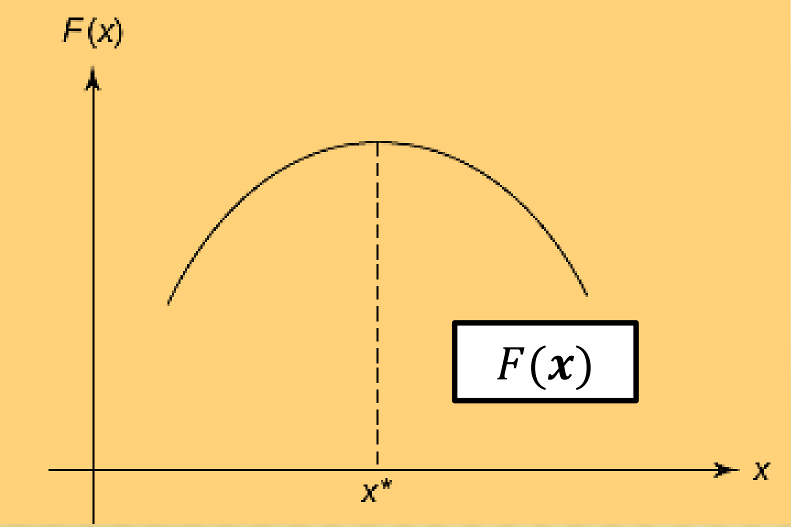
IMPORTANT OBSERVATIONS---CONT.

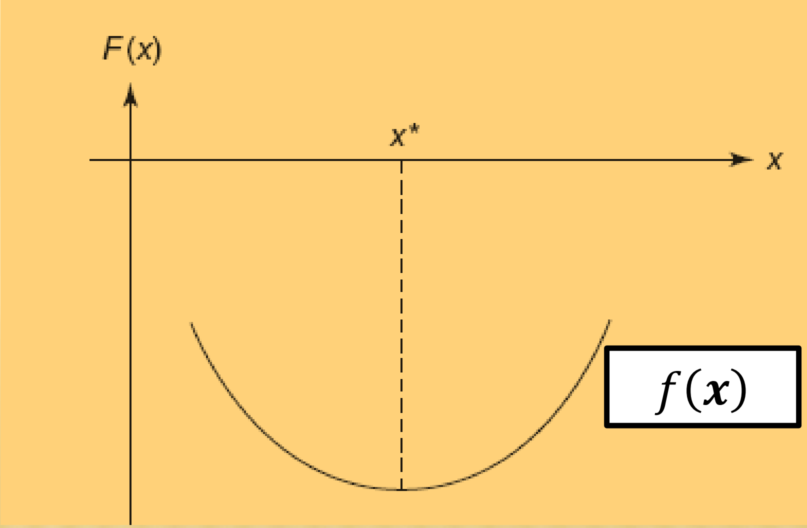
- No restriction on number of inequality constraints

$$p \leq n \quad or \quad p \geq n$$

- General design model treats only minimization problems, let $F(\boldsymbol{x})$ is maximization function

$$f(\boldsymbol{x}) = -F(\boldsymbol{x})$$





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IMPORTANT OBSERVATIONS---CONT.

- Standard design optimization model treats only "≤ type" inequality constraints, let $G_l(\boldsymbol{x}) \geq 0$ is inequality constraint

$$g_l(\boldsymbol{x}) = -G_l(\boldsymbol{x}) \leq 0$$

- For inequality constraints $g_l(\boldsymbol{x}) \leq 0$, $\boldsymbol{x}^*$ is the design point

$g_l(\boldsymbol{x}^*) = 0$	⟹	Active Constraint
$g_l(\boldsymbol{x}^*) < 0$	⟹	Inactive Constraint
$g_l(\boldsymbol{x}^*) > 0$	⟹	Violated Constraint

INTRODUCTION TO OPTIMIZATION THEORY
IMPORTANT OBSERVATIONS---CONT.

- For equality constraints $h_j(\boldsymbol{x}) = 0$, $\boldsymbol{x}^*$ is the design point

$h_j(\boldsymbol{x}^*) = 0$	→	Active Constraint
$h_j(\boldsymbol{x}^*) \neq 0$	→	Violated Constraint

Thank You for Your Attention