ELECTIVE 2
OPTIMAL CONTROL SYSTEMS (ACE 326)

Lecture 2- Introduction to Optimization Theory---Cont.

Ref. 1: Chapters 1&2

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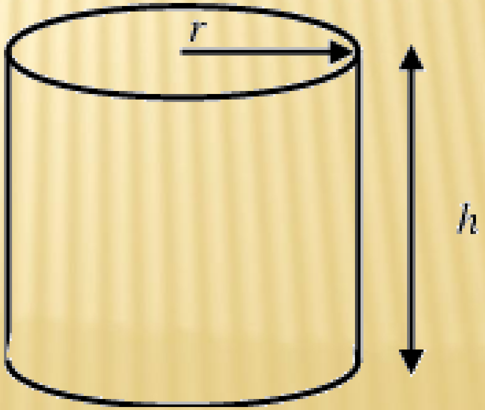
INTRODUCTION TO OPTIMIZATION THEORY
OUTLINES

- **More Applications on Optimization Theory**

INTRODUCTION TO OPTIMIZATION THEORY
OPTIMUM DESIGN APPLICATION

Example 2: **Design a minimum-cost cylindrical tank** **closed at both ends to contain a fixed volume of fluid V. The cost is found to depend directly on the area of sheet metal used.**

Step1: Project/Problem Description



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OPTIMUM DESIGN APPLICATION

Step2: Data & Information Collection

- ✓ The engineering models of this system are:

$$A = 2\pi r^2 + 2\pi rh$$

$$V = \pi r^2 h$$

- ✓ c: is the currency cost per unit area

Step3: Definition of Design Variables

- ✓ $r =$ radius of the tank, unit
- ✓ $h =$ height of the tank, unit

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OPTIMUM DESIGN APPLICATION

Step4: Optimization Criteria

- ✓ The design objective $f(\boldsymbol{x})$ is to minimize the cost

$$f(r, h) = c(2\pi r^2 + 2\pi r h)$$

Step5: Formulation of Constraints

$$V = \pi r^2 h$$

$$r_{min} \leq r \leq r_{max}$$

$$h_{min} \leq h \leq h_{max}$$

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HOW IS OPTIMIZATION?

Summary

The optimization problem has the following features:

1. Single objective
2. Constrained (equality/inequality)
3. Continuous
4. Nonlinear
5. Static

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OPTIMUM DESIGN APPLICATION

Example 3: **A company owns two sawmills and two forests. Each forest can yield up to 200 logs/day, and the cost to transport the logs is estimated at $10/km/log. At least 300 logs are needed daily. The goal is to minimize the total daily cost of transporting the logs.**

Step1: Project/Problem Description

Mill	Distance from forest 1	Distance from forest 2	Mill capacity per day
A	24 km	20.5 km	240 logs
B	17.2km	18 km	320 logs

OPTIMUM DESIGN APPLICATION

Step2: Data & Information Collection

- ✓ Data are given in the table

Step3: Definition of Design Variables

- ✓ x_1 = number of logs shipped from Forest 1 to Mill A
- ✓ x_2 = number of logs shipped from Forest 2 to Mill A
- ✓ x_3 = number of logs shipped from Forest 1 to Mill B
- ✓ x_4 = number of logs shipped from Forest 2 to Mill B

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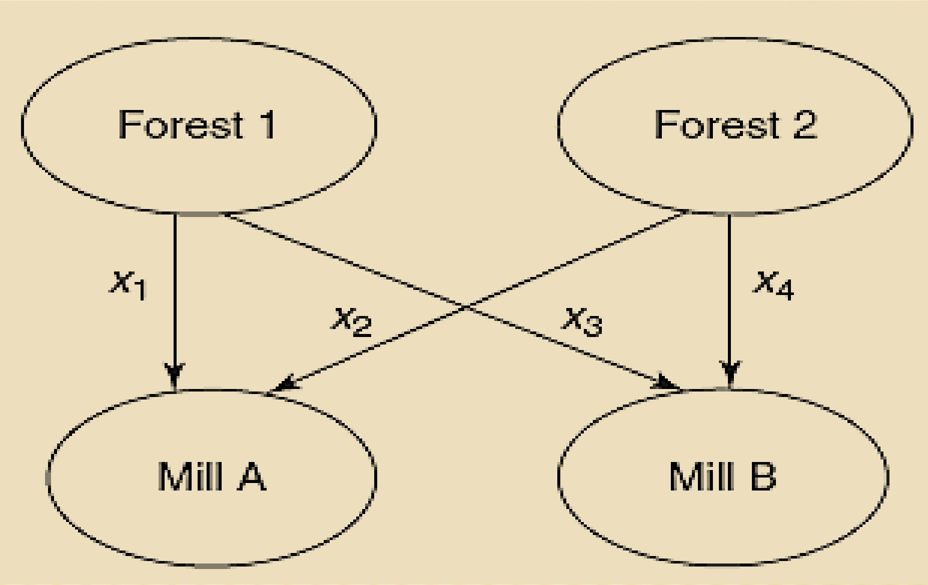
OPTIMUM DESIGN APPLICATION



Step4: Optimization Criteria

- ✓ The design objective $f(\boldsymbol{x})$ is to minimize the cost
  $f(\boldsymbol{x}) = 10(24)\,x_1 + 10(20.5)\,x_2 + 10(17.2)\,x_3 + 10(18)\,x_4$

Step5: Formulation of Constraints

$x_1 + x_3 \leq 200$ (Forest1 production)

$x_2 + x_4 \leq 200$ (Forest 2 production)

$x_1 + x_2 \leq 240$ (Mill A capacity)

$x_3 + x_4 \leq 320$ (Mill B capacity)

$x_1 + x_2 + x_3 + x_4 \geq 300$ (request for logs)

$x_i \geq 0$; i $= 1{:}\,4$

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HOW IS OPTIMIZATION?

Summary

The optimization problem (transportation problems) has the following features:

1. Single objective
2. Constrained (inequality)
3. Discrete (integer numbers)
4. Linear
5. Static

Thank You for Your Attention