ELECTIVE 2
OPTIMAL CONTROL SYSTEMS
(ACE 326)

Lecture 3- Graphical Optimization and Basic Concepts
Ref. 1: Chapter 3

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OUTLINES

- Graphically solve optimization problem
- Plot constraints and identify their feasible/infeasible plane
- Identify the feasible/infeasible region for an optimization problem
- Graphically locate the optimum solution/identify active/inactive constraints
- Identify problems that may have multiple, unbounded, or infeasible solutions

OPTIMIZATION METHODS CLASSIFICATION

Optimization Methods

- **Graphical based**
- **Calculus based**
  - Linear Programming
  - Nonlinear Programming
- **Search based**
  - Particle Swarm
  - Ant colony
  - Genetic algorithm

GRAPHICAL OPTIMIZATION

❑**Example 4:** Plot and identify the feasible/infeasible plane for the following constraints:

$a.\ x_1 \leq x_2$ b. $x_1 + 2x_2 \leq 8$ c. $3x_1 + 4x_2 \leq 12$

Solution:

1. **Write the equation of any inequality**
2. **Write the equation in standard form as**

$$\frac{x_1}{a} + \frac{x_2}{b} = 1$$

3. Draw the boundary line with the previous equation

4. Select a test point, e.g. (0,0), substitute into the inequality. If the test point satisfy the inequality ⟶ feasible plane

GRAPHICAL OPTIMIZATION

1. **Write the equation of any inequality**

a. $x_1 + 2x_2 = 8$

b. $3x_1 + 4x_2 = 12$

c. $x_1 = x_2$

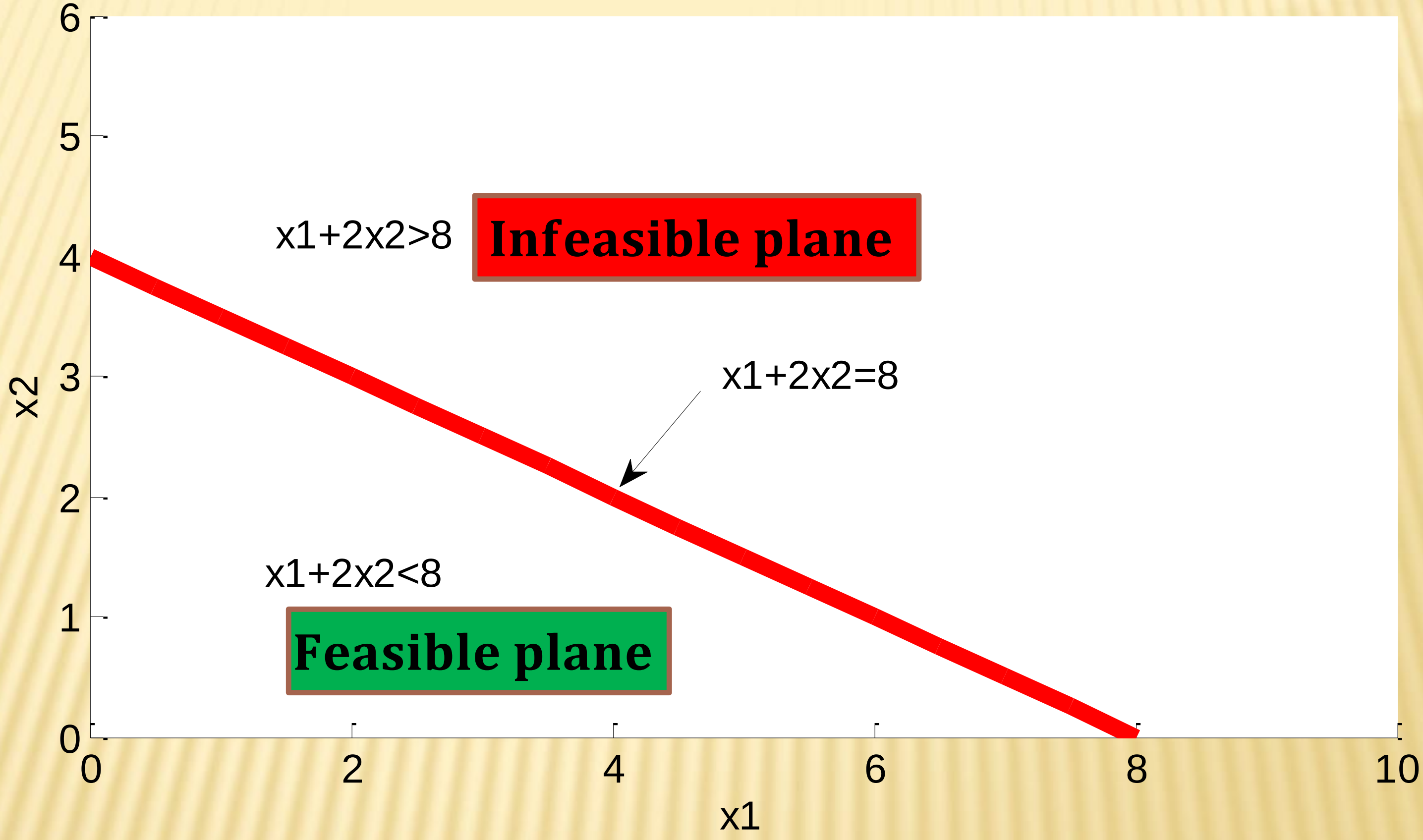
2. **Write the equation in standard form:**

a. $\frac{x_1}{8} + \frac{x_2}{4} = 1$

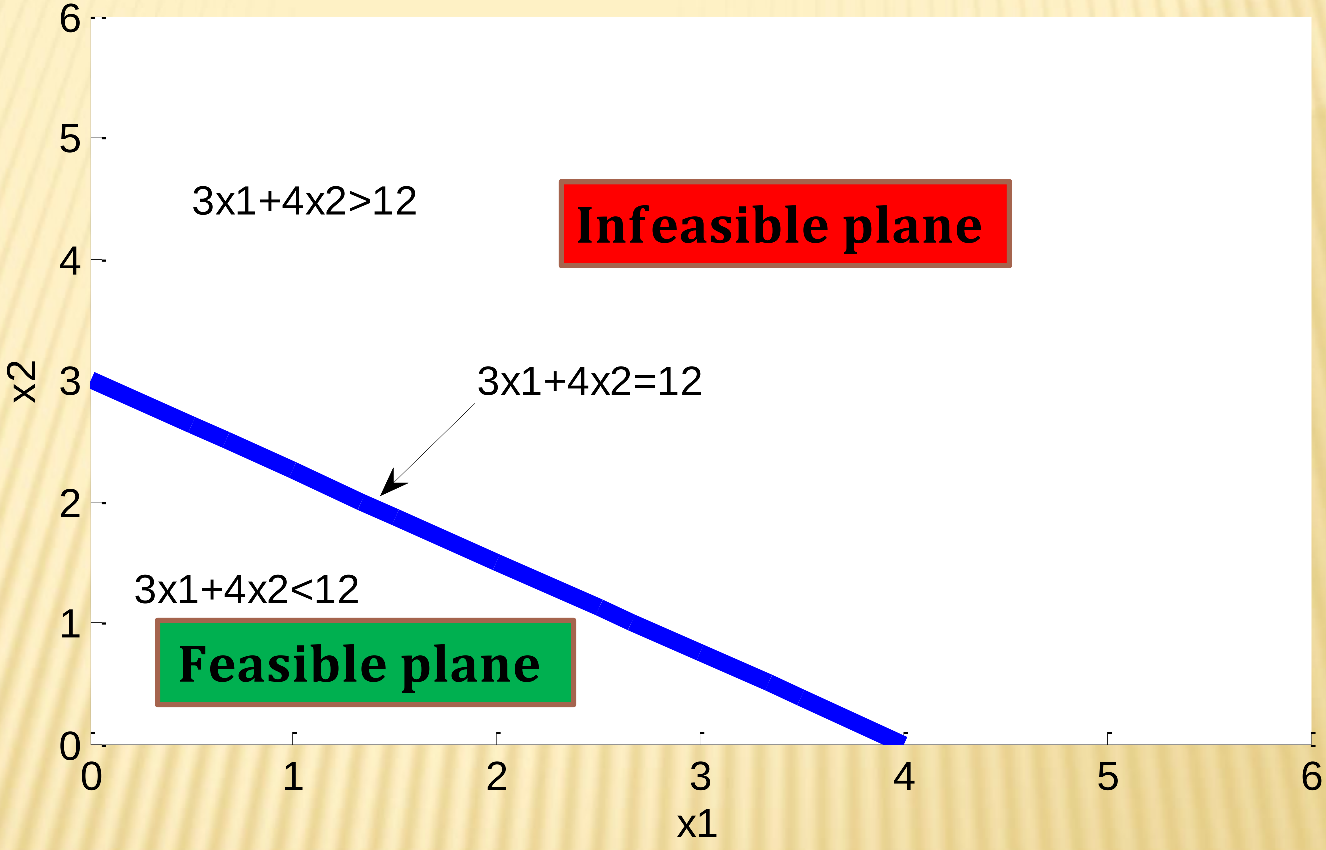
b. $\frac{x_1}{4} + \frac{x_2}{3} = 1$

c. $x_1 = x_2$

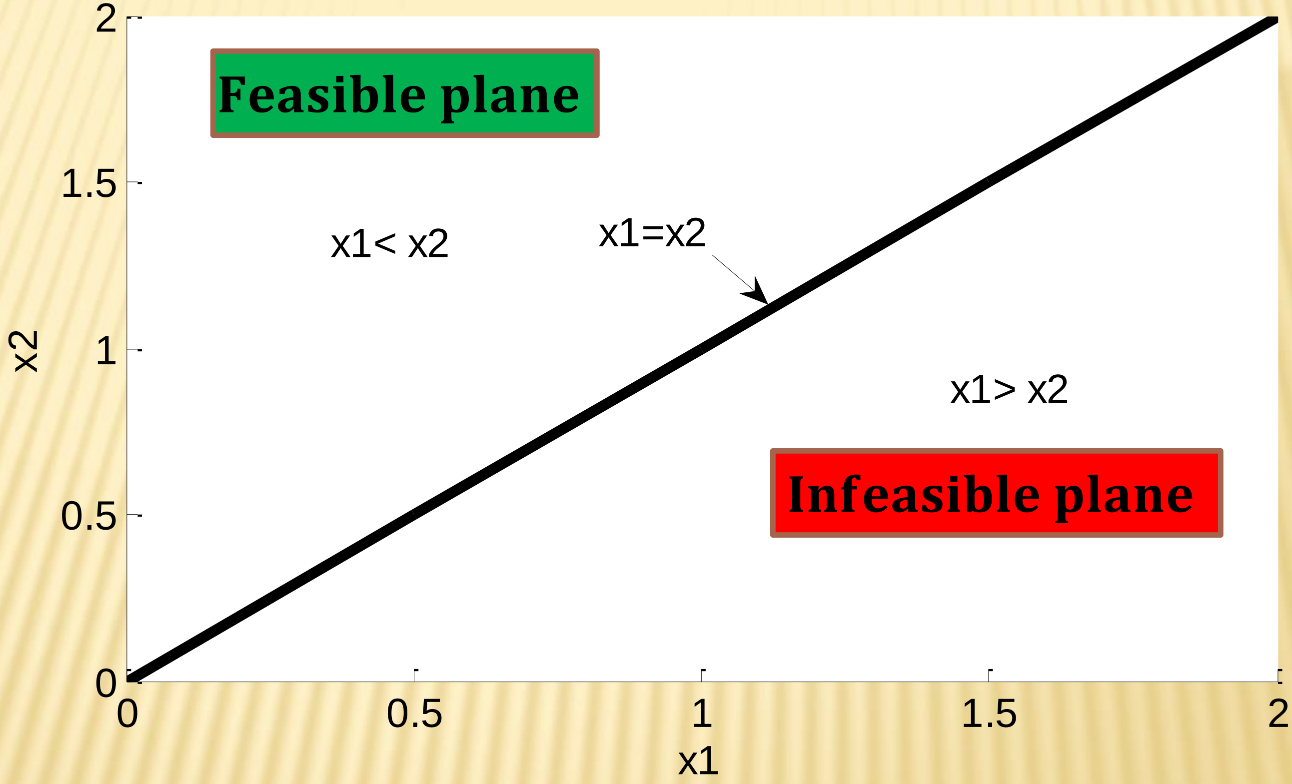
GRAPHICAL OPTIMIZATION



GRAPHICAL OPTIMIZATION



GRAPHICAL OPTIMIZATION



GRAPHICAL OPTIMIZATION

❑**Example 5:** Find the collection of all design points S that satisfies the following constraints:

$a.\ x_1, x_2 \geq 0$ b. $3x_1 + 9x_2 \leq 9$ c. $x_1 + x_2 < 1$

Solution:

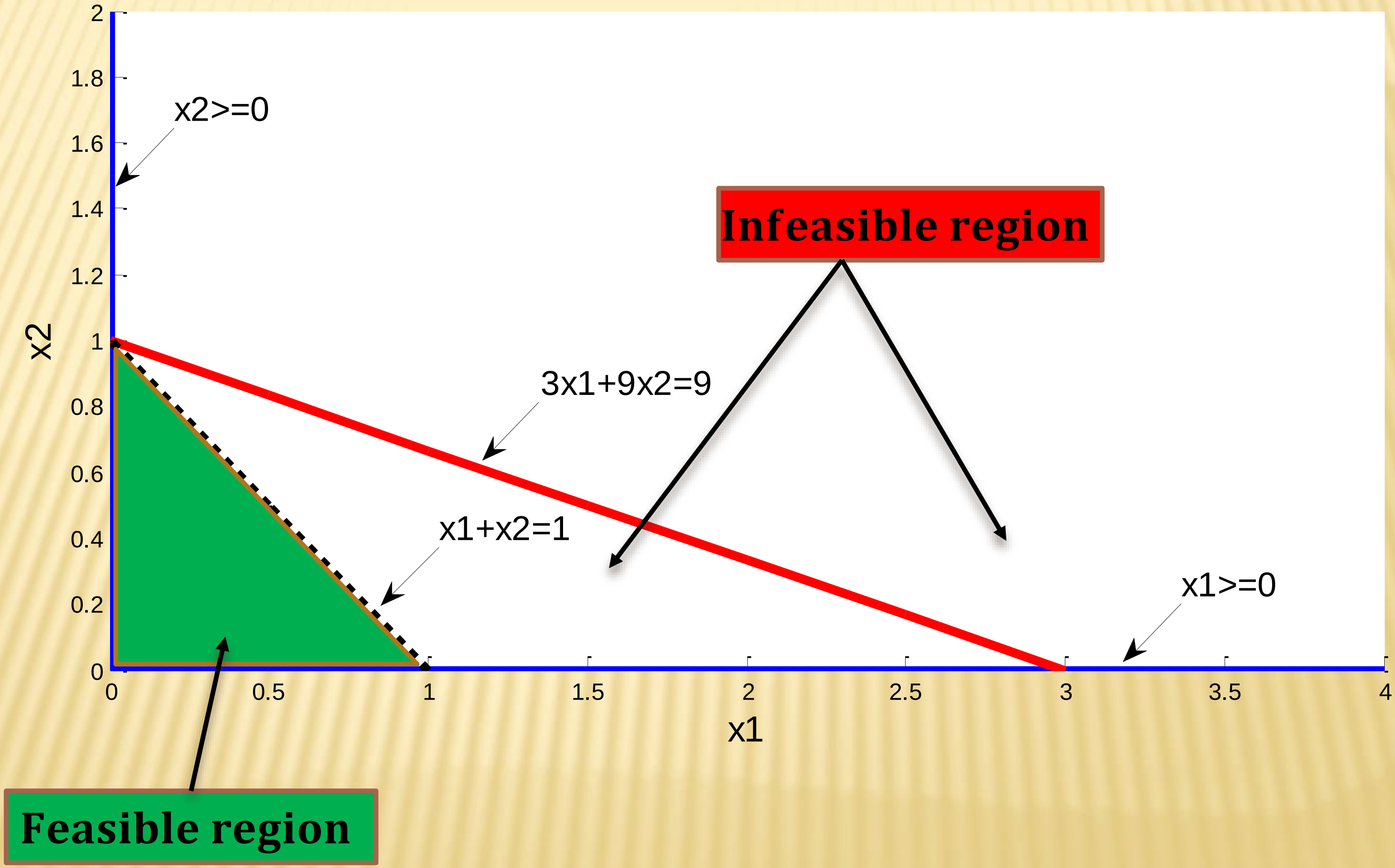
1. **Write the equation of any inequality in standard form:**

a. $x_1, x_2 \geq 0$ ⟶ 1st quadrant

b. $\frac{x_1}{3} + x_2 = 1$

c. $x_1 + x_2 = 1$

GRAPHICAL OPTIMIZATION



GRAPHICAL OPTIMIZATION

Example 6: A company manufactures two machines, A and B. 28 A or 14 B can be manufactured daily. The sales department can sell up to 14 A machines or 24 B machines. The shipping facility can handle no more than 16 machines per day. The company makes a profit of $400 on each A machine and $600 on each B machine. **How many A and B machines should the company manufacture every day to maximize its profit?**

Step1: Project/Problem Description

	Machine A	Machine B	Upper limit
Manufactured	28	14	28 or 14
Sales	14	24	14 or 24
Shipping facility	-	-	16
Profit	$400	$600	-

GRAPHICAL OPTIMIZATION

Step2: Data & Information Collection

- ✓ Data are given in the table

Step3: Definition of Design Variables

- ✓ x_1 = number of A machines manufactured each day
- ✓ x_2 = number of B machines manufactured each day

Step4: Optimization Criteria

- ✓ The design objective $f(\boldsymbol{x})$ is to maximize the profit

$$f(\boldsymbol{x}) = 400x_1 + 600\, x_2$$

GRAPHICAL OPTIMIZATION

Step5: Formulation of Constraints

$x_1 + x_2 \leq 16$ (Shipping facility)

$\frac{x_1}{28} + \frac{x_2}{14} \leq 1$ (Manufacturing)

$\frac{x_1}{14} + \frac{x_2}{24} \leq 1$ (Sales)

$x_1, x_2 \geq 0$ (nonnegative-Integer)

GRAPHICAL SOLUTION

1. **Write the equation of any inequality in standard form:**

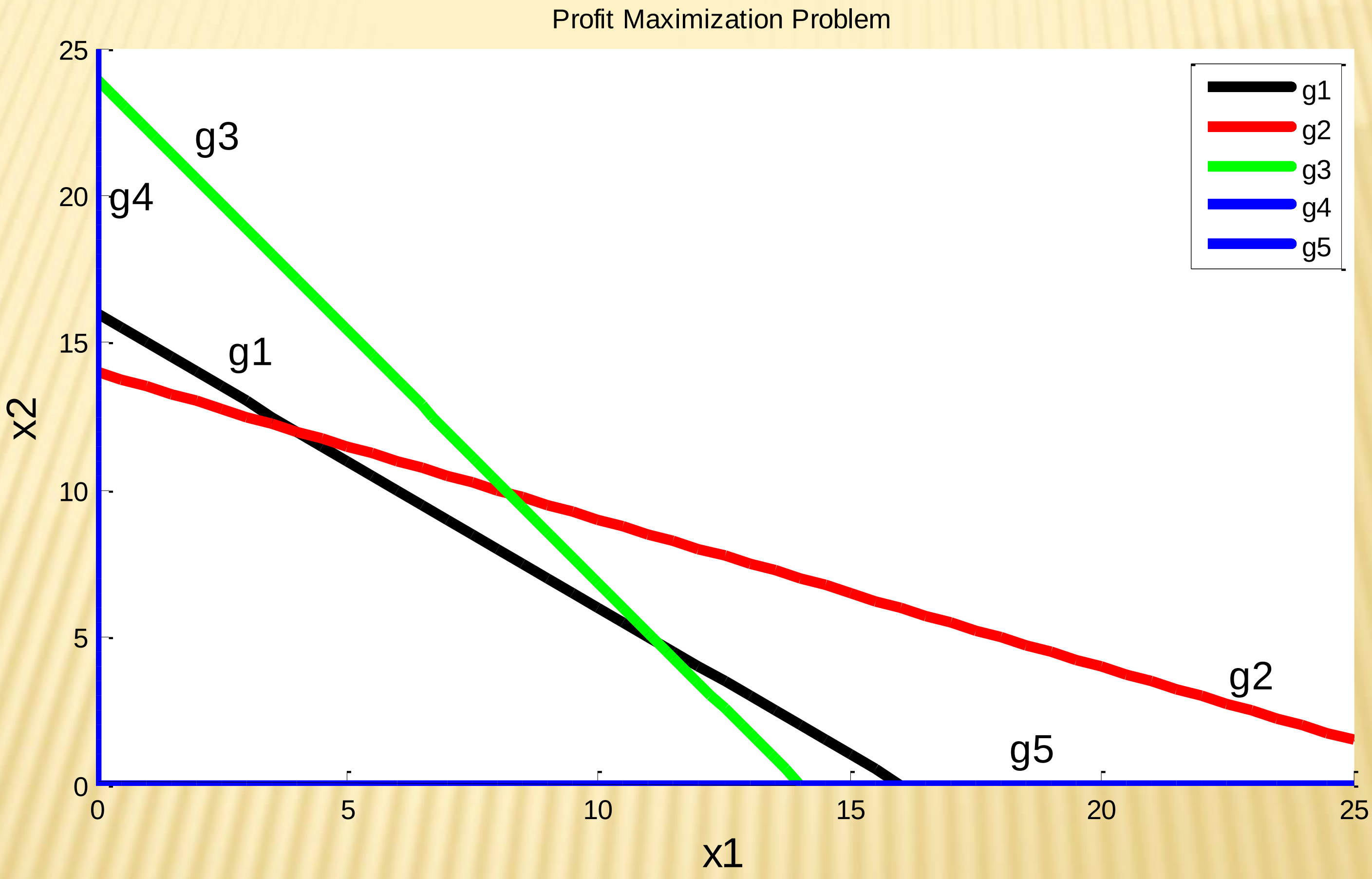
a. $x_1 + x_2 = 16 \; (g1)$

b. $\frac{x_1}{28} + \frac{x_2}{14} = 1 \quad (g_2)$

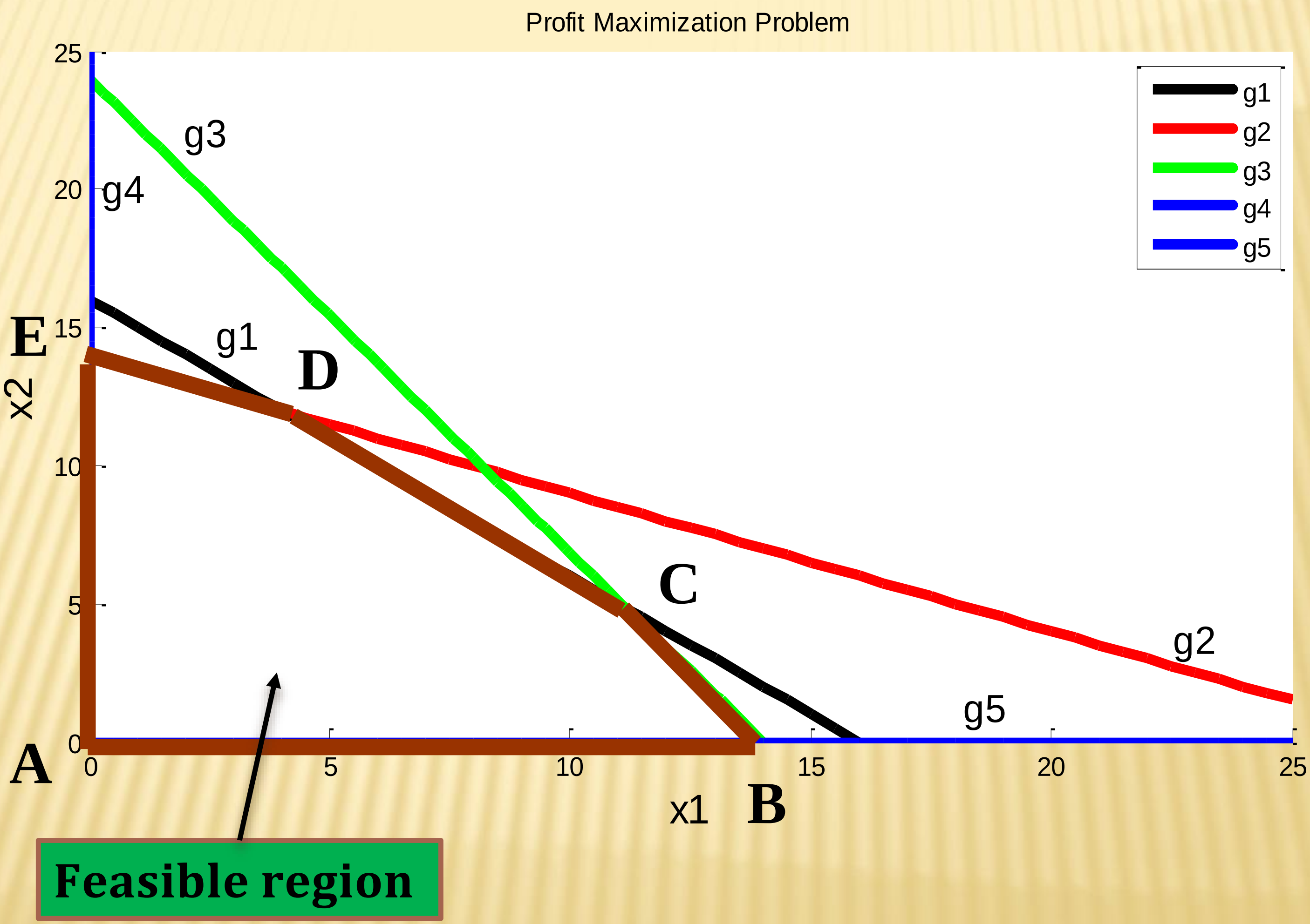
c. $\frac{x_1}{14} + \frac{x_2}{24} = 1 \quad (g_3)$

d. $x_1, x_2 \geq 0 \longrightarrow$ 1^{st} quadrant (g_4 and g_5)

GRAPHICAL SOLUTION



GRAPHICAL SOLUTION



GRAPHICAL SOLUTION

2. **Determine the vertices of the feasible region:**

$$A(0,0),\ B(14,0),\ C(11,5),\ D(4,12),\ and\ E(0,14)$$

3. **Determine the objective function at each vertex:**

$f(\mathbf{0},\mathbf{0}) = 400(0)+600(0)=0$

$f(\mathbf{14},\mathbf{0}) = 400(14)+600(0)=5600$

$f(\mathbf{11},\mathbf{5}) = 400(11)+600(5)=7400$

$f(\mathbf{4},\mathbf{12}) = 400(4)+600(12)=$ 8800 ← **Active constraint**

$f(\mathbf{0},\mathbf{14}) = 400(0)+600(14)=8400$

HOMEWORK

Exercise 1:

$$f(\boldsymbol{x}) = -x_1 - 0.5x_2$$

Subject to $2x_1 + 3\,x_2 \le 12, 2x_1 + x_2 \le 8, -x_1, -x_2 \le 0$

Exercise 2:

$$f(\boldsymbol{x}) = -x_1 + 2x_2$$

Subject to $-2x_1 + x_2 \le 0, -2x_1 + 3x_2 \le 6, -x_1, -x_2 \le 0$

Exercise 3:

$$f(\boldsymbol{x}) = x_1 + 2x_2$$

Subject to $3x_1 + 2x_2 \le 6, 2x_1 + 3x_2 \ge 12$

$$x_1, x_2 \le 5$$

$$x_1, x_2 \ge 0$$

Thank You for Your Attention