1.7. FERMI DIRAC DISTRIBUTION FUNCTION

The energy band divided to energy states, electrons can occupy some of these. According to Pauli statement that, there are many states are unoccupied by electrons and no two electrons has the same energy in the same atom at the same time. Now, what determine the probability that a given energy state occupies by electron? The accurate number of electrons dN having a value of energy in a given range of energy state E+ ΔE at absolute temperature can be expressed by FERMI as:

$$dN = \frac{C\sqrt{E}\,dE}{e^{(E-E_F)/KT}+1} \qquad Eq.1.8$$

Where the constant of proportionality C has the value:

$$C = \frac{4\,\pi\,(2\,m_e)^{3/2}}{h^2} = 6.28\,x10^{27}\ \ elctrons\Big/_{m\ /eV^{3/2}} \qquad Eq.1.9$$

Where, (h) is the Plank's constant, and

E_f is defined as the highest electronic energy at absolute temperature.

At E = E_f, the probability of electron occupancy is 50%.

It is important to note that, at 0K all the charge carriers frozen and lies in the lowest level of the energy gap.

In conduction band, there is large number of states has small probability of occupation, there will be only a few electrons in the conduction band.

In contrast, there a large number of states in valance band, most of them occupied and the probability of occupation is unity, so there will be only a few unoccupied energy states in valance band. The FERMI DIRAC distribution function is symmetrical around the FERMI level, figure (1.18).

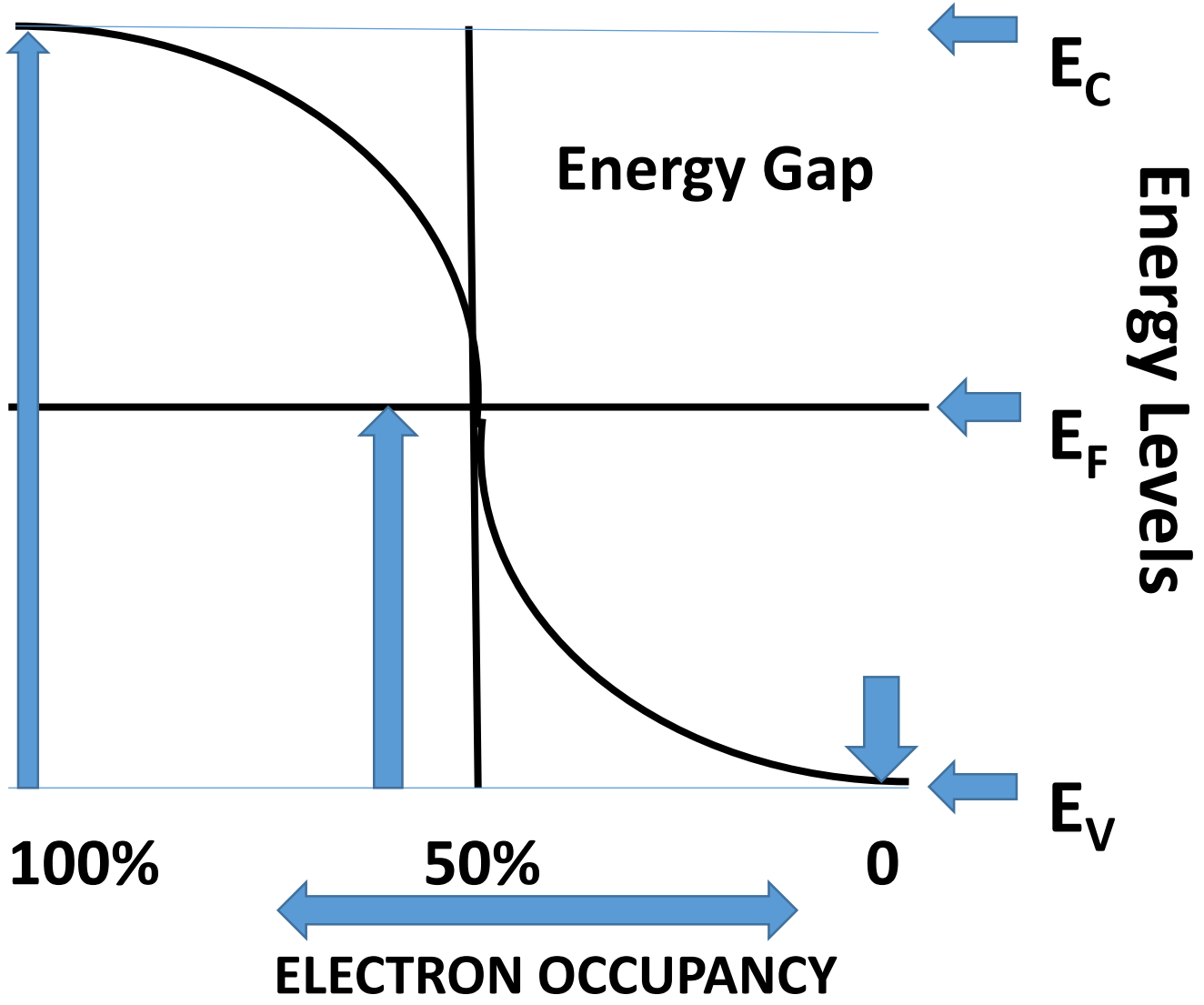


Figure 1-18. Fermi Dirac Distribution Function

If the number of energy states in conduction valance band are the same. In addition, if the number of electrons in conduction and valance band are the same. Then FERMI level must be located in middle of the energy gap in case of intrinsic material. So in intrinsic material n = p and E = E_f and is noted as E_{fi}. In N-Type, the concentration of electrons in conduction band is greater than concentration of electrons in intrinsic material, concentration of holes in conduction band is equal to concentration of holes in intrinsic material. It follows that, in N-Type, FERMI function shifted upward in the energy band picture.

In contrast, in P-Type, FERMI level and FERMI function will both shifted downward in the direction of valance band. The FERMI level serves also as a measure of electronic occupancy in material, and is equal to,

$$E_f = \left[\frac{3\,N}{2\,C}\right]^{3/2}$$

$$= 3.64\,x\,10^{-19}\,N^{3/2} \quad Eq.1.10$$

Where N is the number of valance electrons/m³.

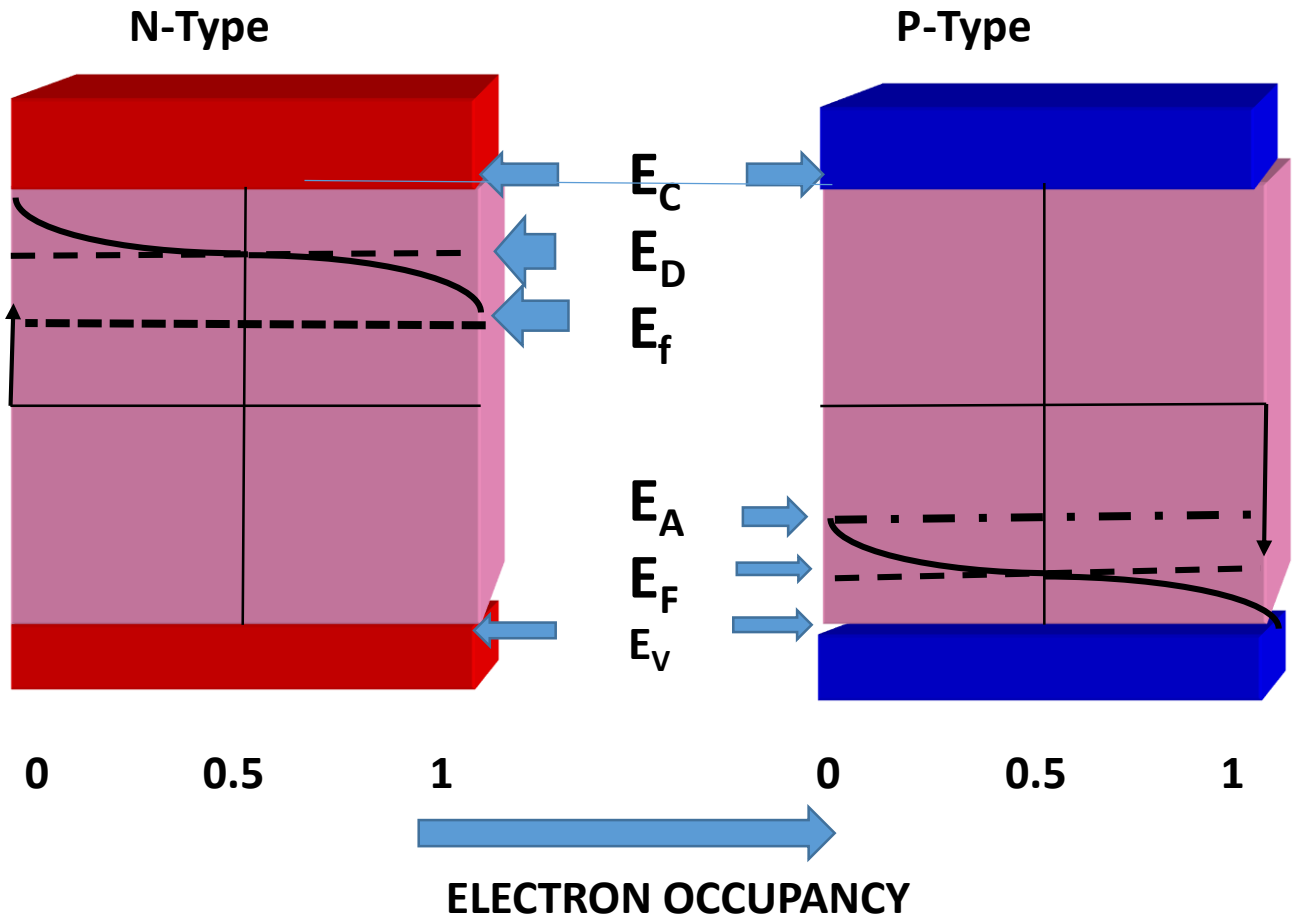


Figure 1.19 Fermi Dirac Distribution Function in N & P-Type Materials

1.7.1. FERMI LEVELS AND CARRIERS DENSITY

The electrical properties of a material depend on presence or absence of electrons in particular energy level. FERMI statistics used to determine the distribution of electrons in any particular energy level. The probability f(E) can be determined by,

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}} \qquad Eq.1.11$$

It should be noted that if E = E_f, then f(E) = 1/2, so that there is 50% chance that an energy state is filled by electron. Figure (1-20) shows the electrons occupancy as function for temperature.

The probability of occupancy increased as the temperature increased and decreased as the temperature decreased. At absolute zero, probability is zero that any level above FERMI be filled, and probability is unity that any level below FERMI will be filled. When we apply this distribution at RT to intrinsic Si which has energy gap 1.1 eV, the energy difference between the two energy states E_c- E_f = 0.55 eV, or about 21 KT (KT = 0.026 eV), in this case E-E_f is so large compared to KT, and Eq.1.11 becomes:

$$f(E) = e^{-(E - E_F)/KT} \qquad Eq.1.12$$

$$1 - f(E) = \frac{1}{1 + e^{-(E_F - E)/KT}}$$

$$Eq.1\ 13$$

Eq.1.13 simplified to:

$$1 - f(E) = e^{-(E_F - E)/KT} \qquad Eq.1\ 14$$

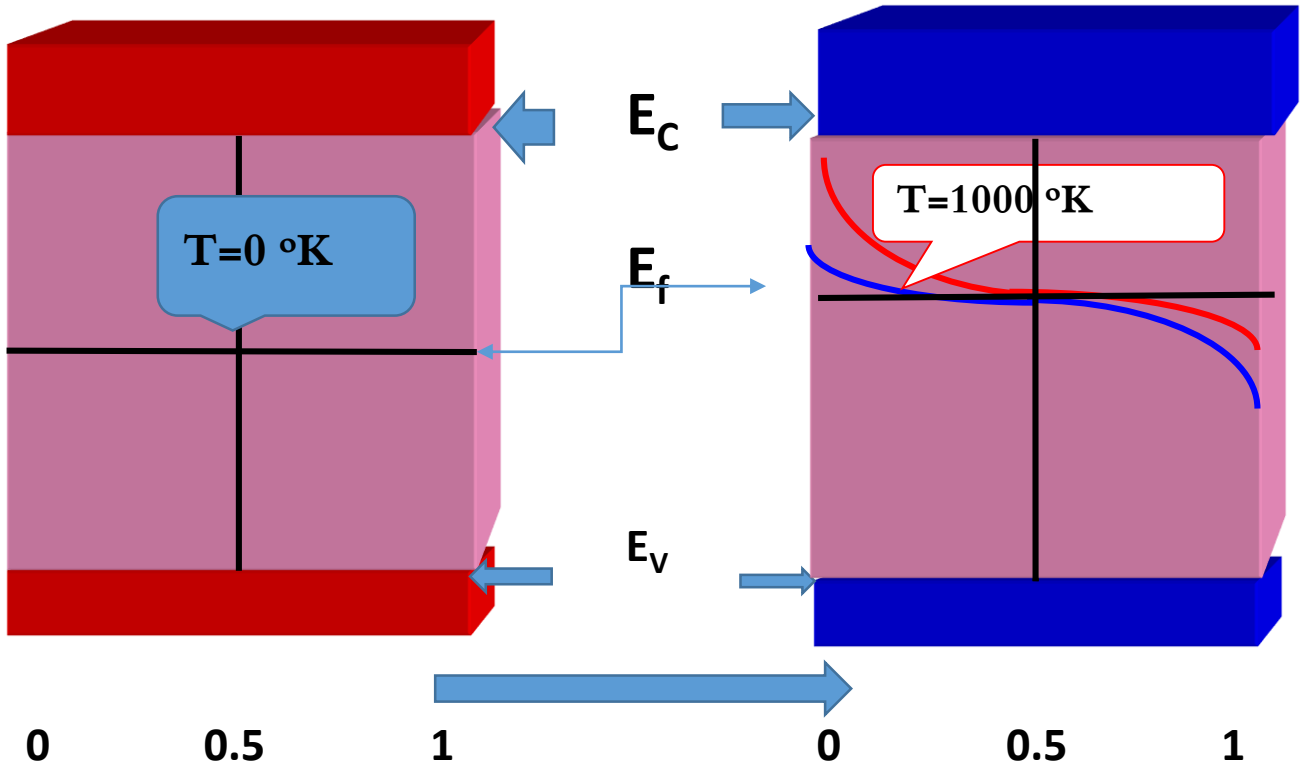


Fig.1.20 FERMI Distribution as Function with Temperature

Eq.1.12 and Eq. 1.14 are used to determine the density of free electrons n and the density of free holes p in the semiconductor material.

If the density of states in which electrons or holes can occupy known, and the probability of occupancy known, the number of free carriers can have determined.

Let N(E) is the number of states/eV/m^3 of the crystal, then the density of states in an energy range E_1 to E_2 is given by:

$$S = \int_{E_1}^{E_2} N(E)\, dE \qquad Eq.1.15$$

Number of electrons is the product of states and the probability of occupancy, so:

$$n = \int_{E_{CB}}^{E_{CT}} f(E)\, N(E)\, dE \qquad Eq.1\ 16$$

Gives: the density of free electrons (n)

$$n = N_c\, e^{-\left(E_c - E_f\right)/KT} \qquad Eq.1.17$$

Where N_c is the effective density of states in the conduction band and equal?

$$N_c = \left[\frac{2\, m_e\, KT}{h^2}\right]^{3/2} \qquad Eq.1.18$$

In addition, the density of free holes p given by:

$$p = N_v\, e^{-\left(E_f - E_v\right)/KT} \qquad Eq.1.19$$

Where N_v is the effective density of states in the valance band and it is equal:

$$N_v = \left[\frac{2\, m_p\, KT}{h^2}\right]^{3/2} \qquad Eq.1.20$$

Where, $N_c = N_v$ only, if the effective masses of electrons and holes are equal, from Eq.1.17 and Eq.1.19, we can calculate the concentration of equilibrium value of free carriers np,

$$np = N_c N_v e^{-(E_c - E_v)/KT} = N_c N_v e^{-(E_G)/KT} \quad Eq.1.21$$

As the energy gap E_g decreases linearly with temperature, so $E_g = E_{go} - \beta$, and Eq.1.21 becomes:

$$np = 2.33\,x\,10^{43} \left[\frac{m_e m_p}{m^2}\right]^{3/2} T^3\, e^{-(E_{Go})/KT}\, e^{\beta/K} \quad Eq.1.22$$

Where E_{go} is the energy gap at 0 K, β is the variation of E_g with temperature. For Si, the equilibrium concentration of intrinsic carriers given by:

$$n\,p = 15\,x\,10^{44}\;T^3\,e^{-(1.21)/KT} \qquad Eq.1.23$$

For intrinsic material where n = p , so the intrinsic concentration is :

$$n_i^2 = n\,p \qquad Eq.1.24$$

In addition, this known as electrical neutrality law. Equations 1.17 and 1.19 for determine electrons and holes concentrations can be applied to both intrinsic and extrinsic materials, wherein intrinsic n = p. So:

$$N_c\,e^{-(E_c-E_f)/KT} = N_v\,e^{-(E_f-E_v)/KT} \qquad Eq.1.25$$

By taking the logarithm of both sides of the equation above,

$$ln\ \frac{N_c}{N_v} = \frac{E_c + E_v - 2E_f}{KT}$$ **From which;**

$$E_f = \frac{E_c + E_v}{2} - \frac{KT}{q}\ ln\ \frac{N_c}{N_v} \qquad Eq.1.26$$

If $N_c = N_v$ at $m_p = m_n$, Eq.1,26 will be simplified to:

$$E_f = \frac{E_c + E_v}{2} \qquad Eq.1.27$$

Solved Examples

Problem (18)

A silicon wafer contains 10^{16} cm^{-3} electrons. Calculate the hole density and the position of the intrinsic energy and the Fermi energy at 300 K. Draw the corresponding band diagram to scale, indicating the conduction and valence band edge, the intrinsic energy level and the Fermi energy level. Use $n_i = 10^{10}$ cm^{-3}.

Solution:

The hole density is obtained using the mass action law:

$$p = \frac{n_i^2}{n} = \frac{10^{20}}{10^{16}} = 10^4\ cm^{-3}$$

The position of the intrinsic energy relative to the mid gap energy equals:

$$E_i - \frac{E_c + E_v}{2} = -\frac{3}{4} KT\ ln\ \frac{m_h^*}{m_e^*} = \frac{3}{4}\ x\ 0.0258\ ln\ \frac{0.81}{1.08} = 5.58\ meV$$

The position of the Fermi energy relative to the intrinsic energy equals:

$$E_F - E_i = KT\ ln\left(\frac{N_d}{n_i}\right) = 0.0258\ ln\ \frac{10^{16}}{10^{10}} = 357\ meV$$

Example (19)

The electron density in silicon at room temperature is twice the intrinsic density. Calculate the hole density, the donor density and the Fermi energy relative to the intrinsic energy. Repeat for $n = 5\ n_i$ and $n = 10\ n_i$. Also repeat for $p = 2\ n_i$, $p = 5\ n_i$ and $p = 10\ n_i$, calculating the electron and acceptor density as well as the Fermi energy relative to the intrinsic energy level.

Solution:

The hole density is obtained using the mass action law: $p = n_i^2/n$

The doping density obtained by requiring charge neutrality $N_d - N_a = n - p$

The Fermi energy obtained from:

$E_F - E_i = kT\ ln(n/n_i)$ Yielding:

	$n = 2\ n_i$	$n = 5\ n_i$	$n = 10\ n_i$
p	$n_i/2$	$n_i/5$	$n_i/10$
Nd - Na	$1.5\ n_i$	$4.8\ n_i$	$9.9\ n_i$
EF - Ei	kT ln(2)	kT ln(5)	kT ln(10)
	$p = 2\ n_i$	$p = 5\ n_i$	p = 10 ni
n	$n_i/2$	$n_i/5$	$n_i/10$
Nd - Na	$-1.5\ n_i$	$-4.8\ n_i$	$-9.9\ n_i$

1.8 EXTRINSIC MATERIALS

In such material, electron concentration is not equal to hole concentration .

In the case of n-type (n > p), where the conduction occurred by electrons in conduction band and Fermi level is shifted upward in the direction of conduction band, and a new level named donor level, E_D, exists.

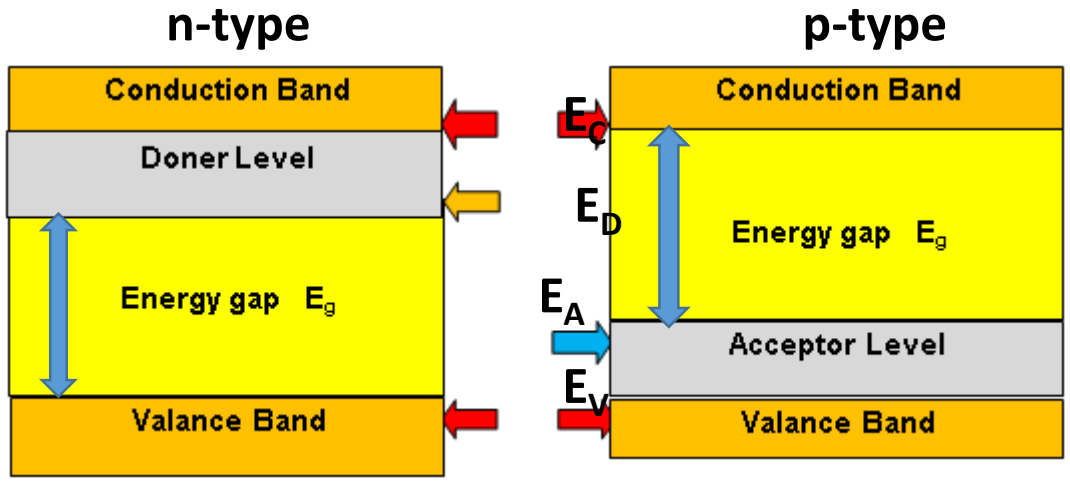


Figure 1-21 Energy Band Representation for Extrinsic Material (a) N-Type (b) P- Type

If (p>n), results p-type material, conduction occurred by holes in valance band and a new level acceptor level, E_A, exists. By adding only one atom from doping material to Si or Ge atoms, the energy required to ionization is 0.01eV for Ge, and 0.05eV for Si and the conductivity is increased by factor of 12. In n-type, number of electrons increased also number of holes decreased below, which was available in intrinsic. Large number of electrons presents due to donor's increases the rate of recombination of electrons with holes, and the same said in case of p-type. For both cases, electrons in n-type and holes in p-type called majorities and the other charges called minorities

1.8.1. CONCENTRATION OF CHARGE CARRIERS IN EXTRINSIC SEMECONDUCTORS

The probability that electrons occupy a donor level can be determined:

$$f(D) = \frac{N_D - N_D^+}{N_D} = 1 - \frac{N_D^+}{N_D} \qquad Eq.1.28$$

The probability that electrons occupy an acceptor level given by:

$$f(A) = \frac{N_A^-}{N_A} \qquad Eq.1.29$$

Where, N_D and N_A are the donor and acceptor levels N_D+, and N_A^- are the ionized donors and acceptor levels.

The concentration of ionized charges can be determined through the probabilities in Fermi distribution function and gives,

$$N_D^+ = \frac{N_D}{g\ x\ e^{(E_f - E_D)/(KT+1)}} \qquad Eq.1.30$$

$$N_A^+ = \frac{N_A}{g\ x\ e^{(E_A - E_f)/(KT+1)}} \qquad Eq.1.31$$

The constant (g) characterizes the structure of levels and equal 1/2. The concentration of electrons in C.B and concentration of holes in V.B is given by:

$$N + N_A^- = P + N_D^+ \qquad Eq.1.32$$

If N_A^- = N_D^+ we have extrinsic material have the same properties of intrinsic. By substituting in Eq.1 .32, we have:

$$N_c\, e^{(E_c - E_f/KT)} + \frac{N_D}{g} e^{-(E_A - E_f/KT)}$$

$$= N_v\, e^{(E_f - E_v/KT)} + \frac{N_D}{g} e^{-(E_f - E_D/KT)} \quad Eq.1.33$$

We supposed that (E_A-E_F) / kT >1; and (E_F- E_D) / KT>1. The solution of Eq.1 .33 is somewhat complicated, for this, we simplify it for both conduction types as follows:

N-Type Material

Where $N_D > N_A$ and p < n, Equation 1.32 simplified to (Eq.1.34), n = N_D, then:

$$N_c\, e^{-(E_c - E_f/KT)} = \frac{N_D}{g} e^{-(E_f - E_D/KT)} \qquad Eq.1.34$$

Or: $\frac{N_c}{N_{D/g}} = e^{-(2E_f - E_D - E_C/KT)}$ From which

$$E_f = \frac{E_D + E_c}{2} - \frac{KT}{2} log^{-g\,(N_C/N_D)} \qquad Eq.1.35$$

If T ➡ 0, then:

$$E_f = \frac{E_D + E_c}{2} \qquad Eq.1.36$$

By substituting Eq.1.35 in Eq.1.17, we can get the concentration of electrons in n-type material.

$$\mathbf{n} = \mathbf{N_c}\, \mathbf{e}^{-(\mathbf{E_c}-\mathbf{E_f}/\mathbf{KT})}$$

$$\mathbf{n} = \mathbf{N_c}\, \mathbf{e}^{-\left[E_c - (\mathbf{E_c}+\mathbf{E_D})/2 + \frac{KT}{2} log \left\{\frac{g\, N_c}{N_D}\right\}/KT\right]}$$

$$n = \frac{N_C}{\sqrt{\frac{g\, N_C}{N_D}}}\, e^{(E_D - E_C)/2KT}$$

$$= \sqrt{\frac{N_D N_C}{g}}\, e^{(-\Delta E_D/2KT)} \quad Eq.1.37$$

P-Type Materials

Where $N_A > N_D$ and $n < p$, Eq.1.32 becomes,

$$P = N_A^- \quad Eq.1.38$$

As we have calculated for p-type:

$$E_f = \frac{E_V + E_c}{2} - \frac{KT}{2} \log e^{g\, N_v/N_A} \quad Eq.1.39$$

If T ⟶ 0, then: The concentration of holes in p-type is,

$$E_f = \frac{E_V + E_A}{2} \quad Eq.1.40$$

$$p = \sqrt{\frac{N_A N_V}{g}}\, e^{(-\Delta E_A/2KT)} \quad Eq.1.41$$

Finally, as we have deduced before in (Eq.1.24), $n_i^2 = np$, n the n-type is majority and it can be written as n_n and p is minority and it can be written as p_n, so we can write (Eq.1 .24) as: $n_i^2 = n_n p_n$ Eq.1.42

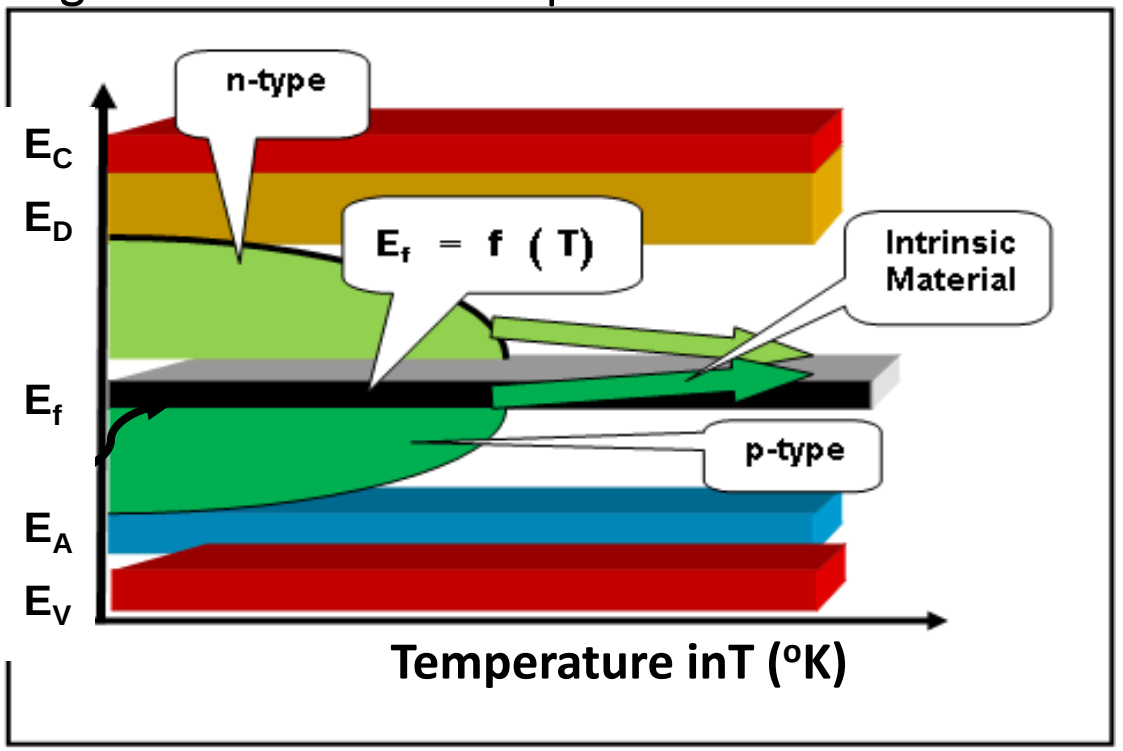
$$p_n = \frac{n_i^2}{n_n} = \frac{n_i^2}{N_D} \qquad Eq.1.43$$

For p-type, p is majority and it written as p_p, and electrons are minority and it written as n_p, so we can write (Eq.1.24) for p-type as: $n_i^2 = p_p n_p$ Eq.1.44

$$n_p = \frac{n_i^2}{P_p} = \frac{n_i^2}{N_A} \qquad Eq.1.45$$

1.8.2. FERMI LEVEL IN EXTRINSIC MATERIAL

Figure 1.22 Effect of Temperature on Fermi Level



The only parameters which changes in equations 1.17 and 1.19 which determine concentrations of n and p when dopant is added is the Fermi energy level E_f which depends on temperature and doping concentrations. If n dopant added to intrinsic material; at a given temperature T all N_D atoms are ionized, and first N_D states in C.B will be filled.

Since E_f is a measure of the probability of occupancy. E_f move closer to C.B to indicate that many energy States in this band filled by donor electrons, as shown in figure 1.21. The same situation leads E_f to move closer to V.B for p-type material. If T increases, all donor atoms are ionized and the concentration of thermally generated electrons in C.B becomes much larger than the concentration of donor electrons. Under this condition, the concentration of holes (p) and electrons (n) become almost equal and the crystal becomes intrinsic as in figure (1-22). We can conclude that as the temperature of the extrinsic material increases E_f moves toward the center of the energy gap.

The exact position of E_f in n-type known if we substitute

$n = N_D$ and Eq.1.17 becomes:

$$N_D = N_C\, e^{-(E_C - E_f)/KT} \qquad \text{Eq. 1.46}$$

And,

$$E_f = E_C - KT \ln \frac{N_C}{N_D} \quad \text{for n-type} \qquad \text{Eq. 1.47}$$

$$E_f = E_V - KT \ln \frac{N_v}{N_A} \quad \text{for p-type} \qquad \text{Eq. 1.48}$$

Note that if $N_A = N_D$, the last two equations are added together, yields:

$$E_f = \frac{E_C + E_v}{2} - \frac{KT}{2} \ln \frac{N_C}{N_v} \qquad \text{Eq. 1.49}$$

Example (24)

A piece of silicon has a resistivity which is specified by the manufacturer to be between 2 and 5 Ohm cm. Assuming that the mobility of electrons is 1400 cm^2/V-sec and that of holes is 450 cm^2/V-sec, what is the minimum possible carrier density and what is the corresponding carrier type? Repeat for the maximum possible carrier density.

Solution

The minimum carrier density obtained for the highest resistivity and the material with the highest carrier mobility, i.e. the n-type silicon. The minimum carrier density therefore equals:

$$n = \frac{1}{q\,\mu_n \rho_{max}} = \frac{1}{1.6\,x\,10^{-19}\,x\,1400x\,5}$$
$$= 8.92\,x\,10^{14} cm^{-3}$$

The maximum carrier density obtained for the lowest resistivity and the material with the lowest carrier mobility, i.e. the p-type silicon. The maximum carrier density therefore equals:

$$p = \frac{1}{q\,\mu_p\,\rho_{max}} = \frac{1}{1.6\,x\,10^{-19}\,x\,450x\,2}$$
$$= 6.94\,x\,10^{15} cm^{-3}$$

Example (26)

Silicon crystal is doped with 5 x $10^{20}/m^3$ atoms per m^3 . The donor level is 0.05 eV from the edge of the conduction band. Taking the band gap to be 1.12 eV, calculate the position of the Fermi level at 200 K.

Solution

The intrinsic carrier concentration obtained from the known carrier concentration in Si at 300 K. As the carrier concentration at 300K is 1.5x$10^{16}/m^3$, the carrier concentration at 200 K is

$$\left(\frac{200}{300}\right)^{3/2} x\,1.5\,x\,10^{16} = 0.82\,x\,10^{16}/\mathrm{m}^3$$

As the doping concentration is much larger than (n_i). we can take,

$$n \approx N_D = 5\,x\,10^{20}/m^3\,, thus$$
$$E_F^n - E_F^i = KT\,ln\,{}^{n}/_{n_i} = 0.183\,eV$$