1.9. Temperature Dependence of Semiconductor Conductivity

Such dependence is one most important in semiconductor . In metals, Conductivity decreases by increasing temperature due to greater frequency of collisions of electrons.

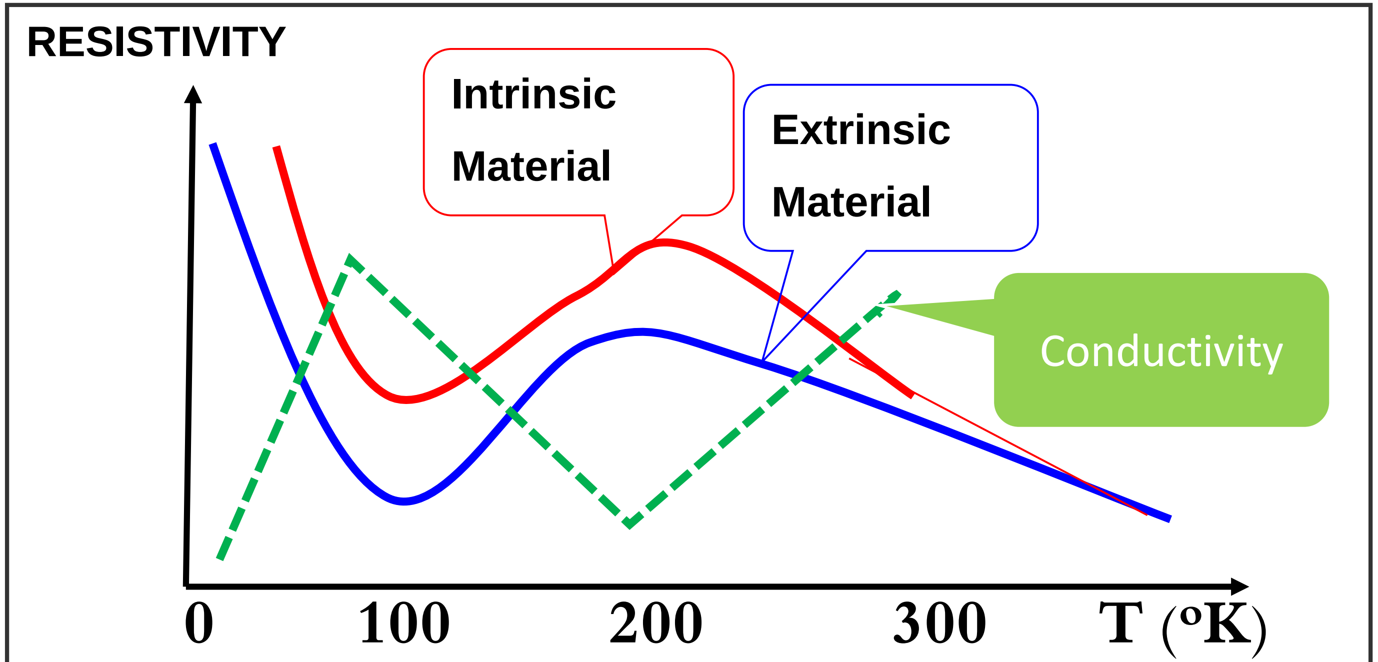


Figure1.23 Temperature Dependence of Semiconductor Conductivity

In semiconductor, in certain temperature ranges the conductivity increases rapidly by increasing temperature

In figure 1.23, at low temperature, the charge carriers are frozen and the resistivity is extremely high, as the temperature raises, increasing fraction of carriers to ionize and the resistivity decreases rapidly because of increasing the ionized charges. When temperature is sufficiently high, most of dopants are completely ionized. The Conductivity begins to decrease and the resistivity is increased again just as in metals. At still higher temperature, there is further sharp decrease in resistivity due to appreciable excitation of all carriers and crossing the energy gap.

Solved Examples

Example (31)

Band gap of Si depends on the temperature as Eg = 1.17 eV – 4.73 × 10^{-4} T^2 / T + 636 . Find a concentration of electrons in the conduction band of intrinsic (undoped) Si at T = 77 K if at 300 K $n_i = 1.05 \times 10^{10}$ cm^{-3}.

Solution:

$$n_i^2 = N_c N_v \, exp\left(-\frac{E_g}{KT}\right)$$

$$\approx T^3 \exp\left(-\frac{E_g}{KT}\right)$$

Therefore,

$$n_1(T_2)$$

$$= n_1(T_1)\left(\frac{T_2}{T_1}\right)^{3/2} \exp\left(-\frac{E_g T_2}{2\,K\,T_2} + \frac{E_g T_1}{2\,K\,T_1}\right)$$

Putting the proper values in the formula we obtain that $n_i(77K) \approx 10^{-20} cm^{-3}$.

Example (32)

Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at 300, 400, 500 and 600 K.

Solution

The intrinsic carrier density in silicon at 300 K equals:

$$n_i(300K) = \sqrt{N_c N_v}\, exp\left(\frac{-E_g}{2\,KT}\right)$$

$$= \sqrt{2.81\, x\, 10^{19}\, x\, 1.83\, x\, 10^{19}}\, exp\left(\frac{-1.12}{2\, x\, 0.0258}\right)$$

$$= 8.72\, x\, 10^9 m^{-3}$$

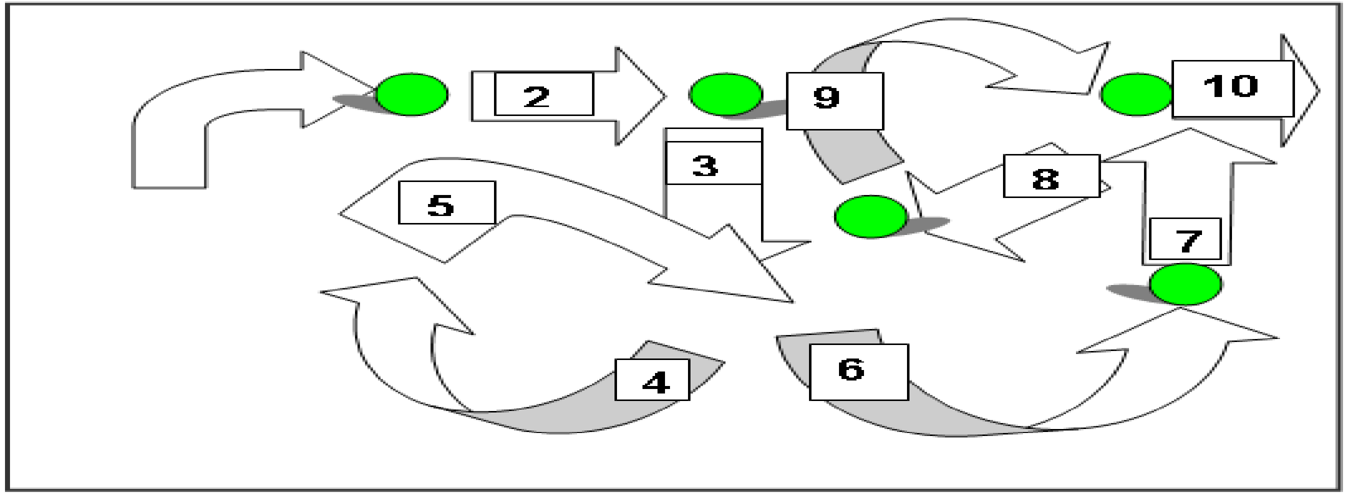
Similarly, one finds the intrinsic carrier density for germanium and gallium arsenide at different temperatures, yielding:

	Germanium	Silicon	Gallium Arsenide
300 K	2.02 x 10^{13}	8.72 x 10^{9}	2.03 x 10^{6}
400 K	1.38 x 10^{15}	4.52 x 10^{12}	5.98 x 10^{9}
500 K	1.91 x 10^{16}	2.16 x 10^{14}	7.98 x 10^{11}
600 K	1.18 x 10^{17}	3.07 x 10^{15}	2.22 x 10^{13}

1.10. The Concept of Mobility

When an electron introduced without subjected external electric field at absolute temperature in a perfect semiconductor sample. The electron will move freely random through the crystal as shown in figure 1.24.

Figure1.24. Random Electron Path at no Field



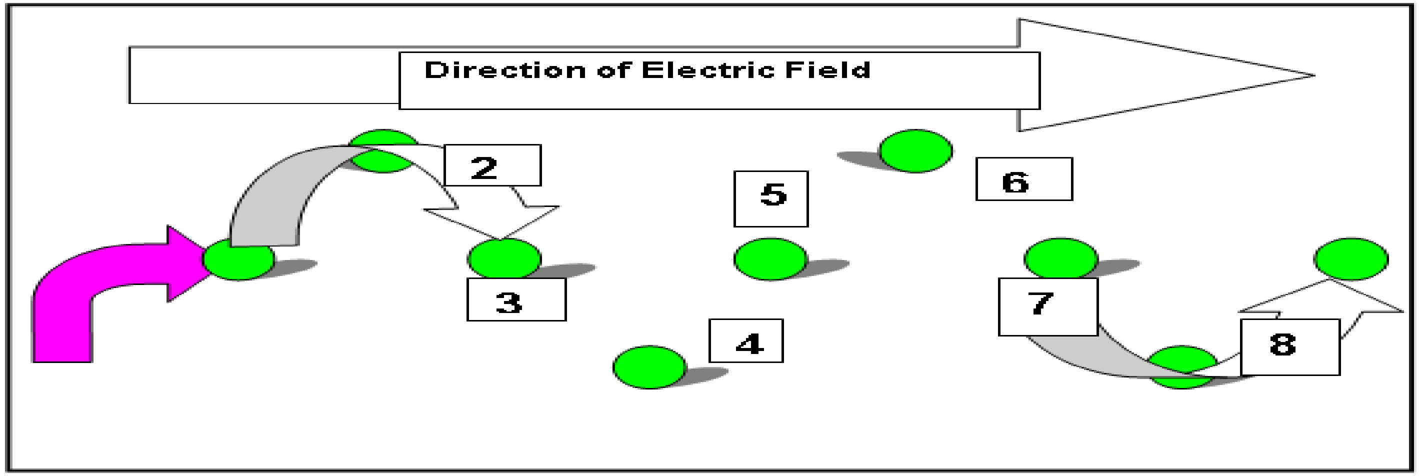


Figure1.25. Directed Electron Path under Field

This is a general property associated with perfect periodic structures. At room temperature, valance electrons are liberated by random motion through the crystal. The random velocity of an electron at room temperature is about 10^7 cm/sec. The mean free path of the electrons between two collisions is in the order of 10^{-5} about 500 times the distance between neighboring atoms. The average time between collisions is about 10^{-12} second. Under thermal equilibrium condition, the random motion of electrons leads to zero current in any direction. Thermal vibration introduced treated as a particle (phonons). Its collisions with electrons and holes called scattering. The scattering phenomena increased as temperature increases for a maximum scattering velocity.

If we apply electric field to the crystal, the electrons gain a force qε toward the positive pole as shown in fig.1.25, and moves with acceleration given by:

$$a = -\frac{q\,\varepsilon}{m} \qquad Eq.1.50$$

After time (τ) , the electrons suffer of collisions decreases velocity (ϑ) between each two consecutive collisions as $\Delta\,\vartheta$ = a τ, produces what is called drift velocity (ϑ_d) .

$$\vartheta_d = \Delta\,\vartheta = \mu\,\varepsilon \qquad Eq.1.51$$

The term (qτ/m) gives the mobility of the charge carriers, then (Eq.1.51) becomes:

$$\vartheta_d = \mu\,\varepsilon \qquad Eq.1.52$$

Normally, $\mu_n > \mu_p$ for any material. For example, in silicon μ_n = 1800, μ_p = 400 cm^2/V.sec, and in germanium we find that μ_n = 3800, μ_p = 1800 cm^2/V.sec.

1.10.1. FIELD DEPENDENCE ON MOBILITY

By increasing the subjected field, the drift velocity increases, due to the increasing of kinetic energy of the electrons. When electric field is higher increased, a critical value developed and reaches maximum scattering velocity; this velocity is not more increased by increasing electric field, **figure 1.26.**

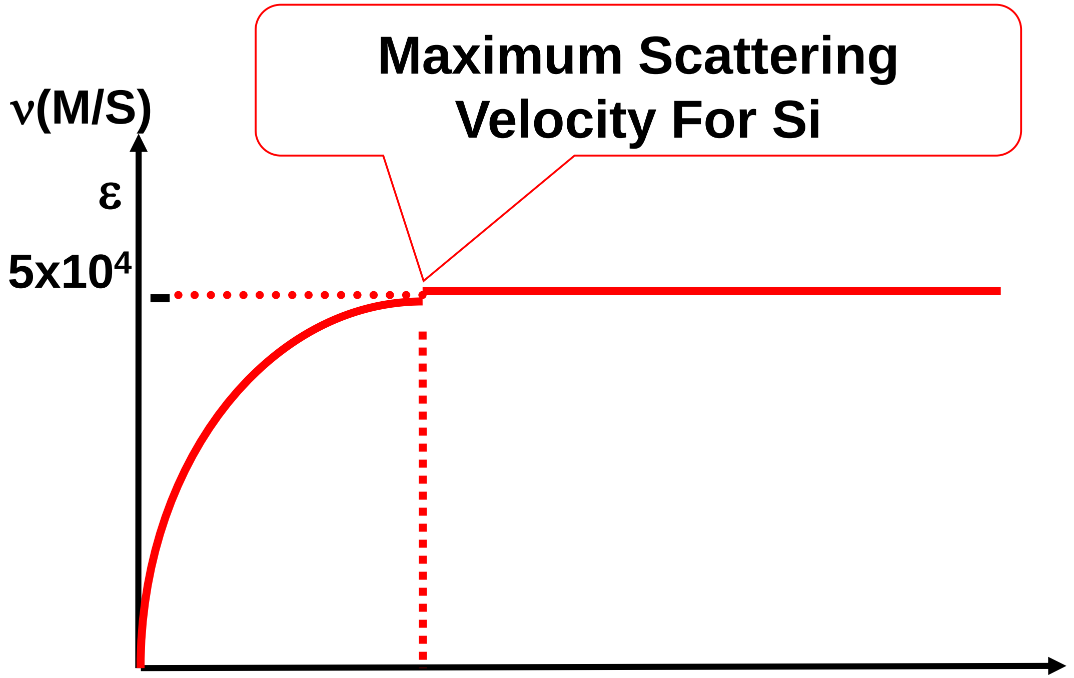


Figure 1.26 Field Dependence on Mobility

Solved Examples

Example (36)

Electrons in silicon carbide have a mobility of 1400 cm^2/V-sec. At what value of the electric field do the electrons reach a velocity of 3×10^7 cm/s? Assume that the mobility is constant and independent of the electric field. What voltage is required to obtain this field in a 5 micron thick region? How much time do the electrons need to cross the 5 micron thick region?

Solution:

The electric field obtained from the mobility and the velocity:

$$\varepsilon = \frac{\mu}{v} = \frac{1400}{3\ x\ 10^7} = 30\ kV/cm$$

Combined with the length one finds the applied voltage.

$V = \varepsilon L = 30{,}000 \times 5 \times 10^{-6} = 0.15$ V

The transit time equals the length divided by the velocity:

$t_r = L/v = 5 \times 10^{-6} / 3 \times 10^7 = 16.7$ ps

Example (37)

The resistivity of a silicon wafer at room temperature is 5 Ωcm. What is the doping density? Find all possible

Solution:

starting with a initial guess that the conductivity is due to electrons with a mobility of 1400 cm^2/V-s, the corresponding doping density equals:

$$N_D = n = \frac{1}{q\,\mu_n \rho} = \frac{1}{1.6\ x\ 10^{-19}\ x\ 1400\ x\ 5} = 8.9\ x\ 10^{14} cm^{-3}$$

The mobility corresponding to this doping density equals

$$\mu_n = \mu_{max} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N_D}{N_\rho}\right)^a} = 1366\ cm^2/vs$$

Since the calculated mobility is not the same as the initial guess, this process must be repeated until the assumed mobility is the same as the mobility corresponding to the calculated doping density, yielding:

$N_d = 9.12 \times 10^{14}\ cm^{-3}$ and $\mu_n = 1365\ cm^2/V$-s

For p-type material one finds:

1.10.2. RECOMBINATION CENTERS MECHANISM

It is an electronic state in the energy gap of semiconductor materials. These states considered as imperfections in the crystal. Metallic impurities are capable of introducing such energy states in the energy gap. The recombination rate is affected by volume of impurities, and surface imperfections. There are three main factors affecting the mobility of charge carriers in semiconductors, they are:

1.10.2.1. Temperature: As temperature increases, the thermal kinetic energy increases the vibration of atoms and the charge carriers suffer from Collisions, the dependence of mobility in temperature given by:

$$\mu_L = KT^{-3/2} \quad Eq. 1.53$$

1.10.2.2. Impurities: The scattering of charge carriers results from the presence of ionized donors or acceptors or impurities. This charged centers will deflect the motion of carriers by the electrostatic forces between two bodies, so the density of such centers affect the velocity; it is also being noted that, the impurity scattering decreases as temperature increases.

$$\mu_I = \frac{KT^{3/2}}{N_I} \qquad Eq. 1.54$$

Where N_I , is the density of ionized centers. Appreciable reduction of mobility results. For example, in germanium the hole mobility falls to 900 $cm^2/V.S.$ (Half its maximum) when the resistivity is 0.06Ω. Materials in this order of impurity are actually employing in semiconductor devices.

1.10.2.3. Dislocations:

Dislocation is atomic misfit, where atoms not probably arranged, so it has a considerable role of scattering carriers. For example, in germanium, the dislocations behave as acceptors, and the mobility affects by:

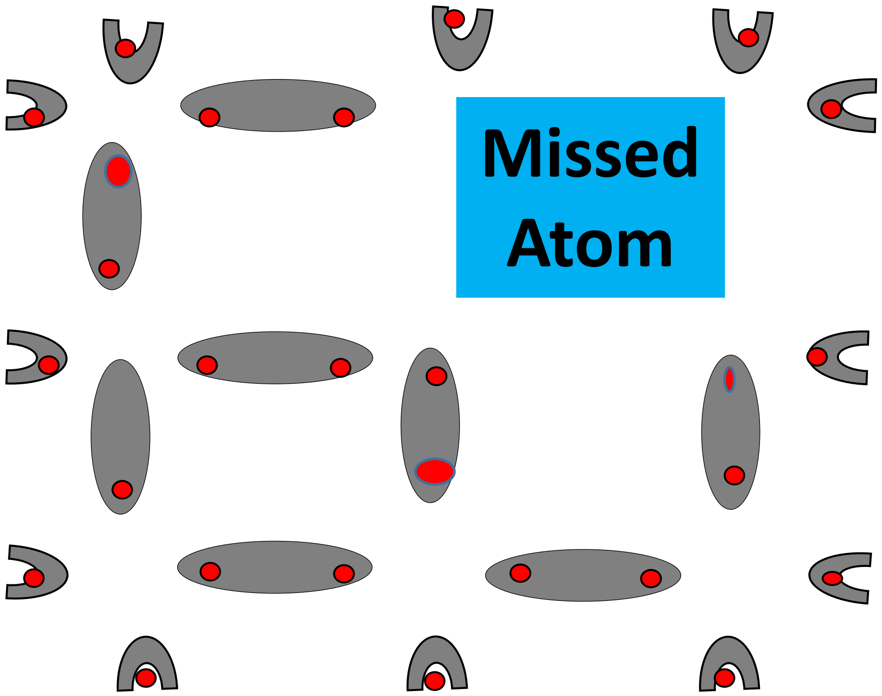
$$\mu_D = KT \qquad Eq.1.55$$

Now if we combine these three parameters, we have a general expression to determine such effects in mobility of charge carriers.

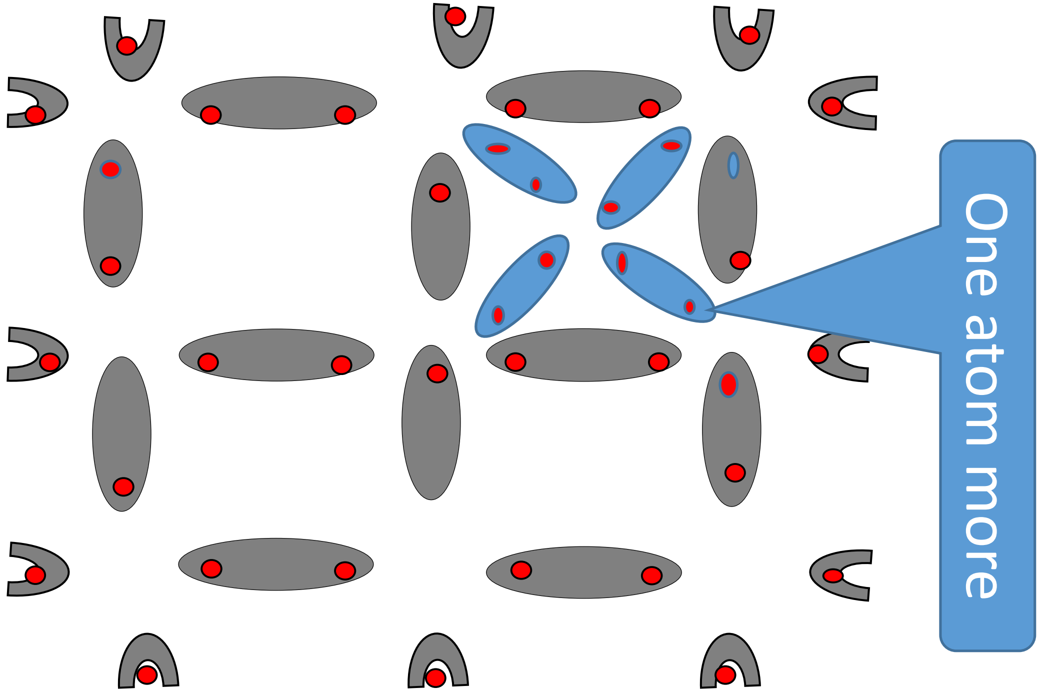
$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I} + \frac{1}{\mu_D} = \alpha_L T^{3/2} + \alpha_I T^{-3/2} + \alpha_D T^{-1} \; Eq.1.56$$

Only the first two terms are normally important, because they are depending more on temperatures.

Vacancies (missed) atom in Covalent Bonding in a Semiconductor Crystal



Intertitles atom in the Covalent Bonding in a Semiconductor Crystal



Solved Examples

Example (38)

Calculate the electron and hole densities in an n-type silicon wafer (N_d = 10^{17} cm^{-3}) illuminated uniformly with 10 mW/cm^2 of red light (E_{ph} = 1.8 eV). The absorption coefficient of red light in silicon is 10^{3} cm^{-1}. The minority carrier lifetime is 10 ms.

Solution

The generation rate of electrons and holes equals:

$$G_n = G_p = \propto \frac{P_{OPT}}{E_{ph}\, q} = 10^{-3}\, \frac{10^{-2}}{1.8\, x\, 1.6\, x\, 10^{-19}} = 3.5\, x\, 10^{23}\, cm^{-3} s^{-1}$$

Where, α is absorption coefficient, P_{opt} illumination power , E_{pt} is the red light , where the photon energy was converted into Joules. The excess carrier densities then obtained from:

$$\delta_n = \delta_p = \tau_p\, G_p = 10\, x\, 10^{-3}\, x\, 3.5\, x\, 10^{23} = 3.5\, x\, 10^{21}\, cm^{-3}$$

The excess carrier densities then obtained from: So that the electron and hole densities equal:

$$n = n_o + \delta_n = 10^{17} + 3.5\, x\, 10^{21} = 3.5\, x\, 10^{21} cm^{-3}$$

Example (39)

What are the approximate thermal velocities of electrons and holes in silicon at room temperature?

SOLUTION:

Assume T = 300 K and recall m_n = 0.26 m_0.

$$Kinetic\ energy = \frac{1}{2}\, m_n\, V_{th}^2 = \frac{3}{2}\, KT$$

$$V_{th} = \sqrt{\frac{3\, KT}{m} = \left[\left(3\, x\, 1.38\, x\, 10^{-23}\, {}^{J}/_{K}\right)\, x\, \left(\frac{300\, K}{0.26}\, x\, 9.1\, x\, 10\right.\right.}$$

$$= 2.3\, x\, m^{5}\, {}^{m}/_{s} = 2.3\, x\, 10^{7}\, {}^{cm}/_{s}$$

Note that 1 J = 1 $kg\cdot m_2/s_2$. Using m_p = 0.39 m_0 instead of m_n, one would find the hole thermal velocity to be 2.2 × 10^7cm/s. Therefore, the typical thermal velocity of electrons and holes is 2.5 × 10^7cm/s, which is about 1000 times slower than the speed of light and 100 times faster than the sonic speed.

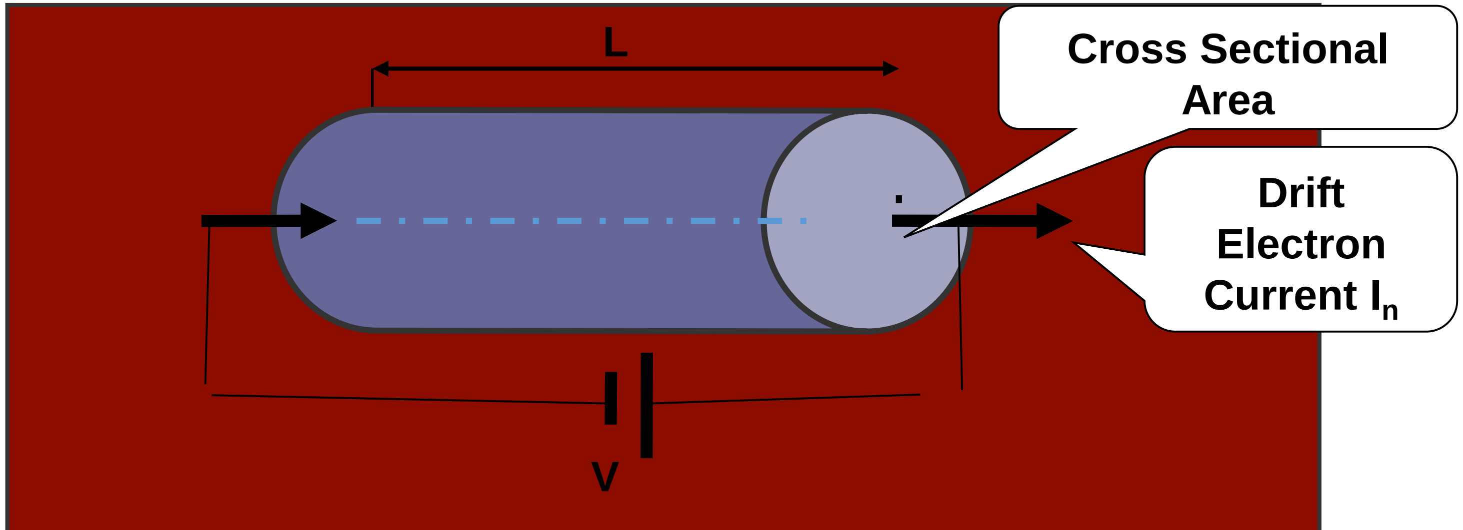
1.11. THE DRIFT CURRENT

Transport of charge carriers under electric field produces a drift current. The current flow in a sample of figure (1.27), having electron concentration (n) given by:

$$I_n = -q\, n\, \vartheta_d\, A \qquad Eq. 1.57$$

Substitute in the drift velocity, gives:

$$I_n = -q\, n\, (\mu_n\, \varepsilon)\, A \qquad Eq. 1.58$$



.Figure 1-27. Current Conduction in a Semiconductor Bar

we know, $\varepsilon = \frac{V}{L}$, $R = \frac{V}{I}$, $\sigma = qn\,\mu_n$

We can find the resistance of the sample due to the electron component as:

$$R_n = \frac{\varepsilon\, L}{q\, A\, n\, \mu_n\, \varepsilon} = \frac{L}{q\, A\, n\, \mu_n} = \rho_n \frac{L}{A} \qquad Eq. 1.59$$

The same steps can be done to determine the drift current due to the hole component in p-type material,

$$I_p = q\, p\, (\mu_{np}\, \varepsilon)\, A$$

$$\sigma_p = qp\, \mu_p$$

$$\rho_p = \frac{1}{qp\mu_p} \qquad Eq. 1.60$$

In semiconductor, both carriers are included, so:

$$\sigma_T = \sigma_n + \sigma_p = q\left(n\,\mu_n + p\,\mu_p\right)$$

$$I_T = q\,A\,\varepsilon\left(n\,\mu_n + p\,\mu_p\right) \quad Eq.1.61$$

In intrinsic, $n = p = n_i$, then,

$$\sigma_j = q\,n_i\left(n\,\mu_n + p\,\mu_p\right) \quad Eq.1.62$$

Note that $\mu_n > \mu_{p,}$ so in an intrinsic material the electrons contribute more to conductivity than holes. It seems that the conduction is due to electrons, so the conduction in intrinsic material considered as in n-type material. For extrinsic material, the conductivity given by:

$$\sigma_n = q\,n_i\,\mu_n\ where\ n_n > p_n\ \&$$

$$\sigma_p = q\,p\,\mu_p\ here \quad p_p > n_p$$

1.12. THE DIFFUSION CURRENT

Transport of charges in semiconductors called diffusion. In a semiconductor bar, the concentration of charge carrier is not uniform. Diffusion of electrons or holes results from their movement from higher concentration to lower concentration with gradient d/dx where concentration of carriers is not distributed uniform and varies with distance x. sectional area A is,

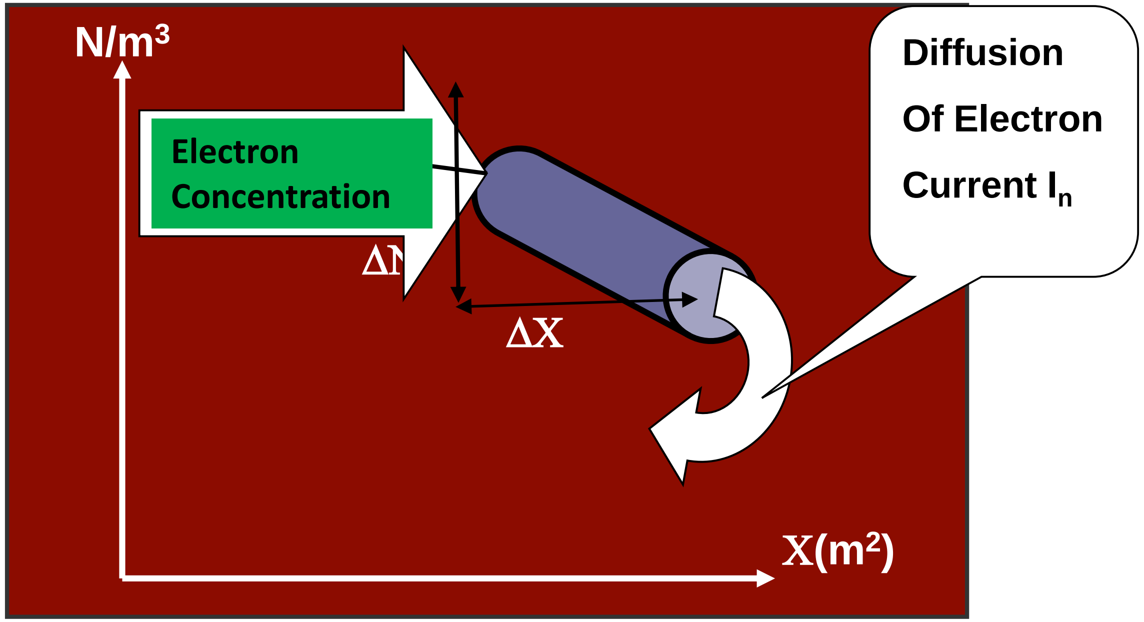


Figure 1.28
Electron Diffusion Gradient

Charges (electrons or holes) are in random motion because of their thermal energy. Their motion gives rise to a current flow known as diffusion current as shown in figure 1.28, diffusion flux obeys Fick's first law:

$$F = -D \frac{\partial N}{\partial x} \qquad Eq. 1.63$$

F is flux of carriers and defined as the number of carriers passing through m^2/sec. Net transport of charges across the surface constitutes current , it is proportional to the concentration gradient. Current passing through cross

$$I = q\,A\,F \qquad Eq. 1.64$$

So that, the diffusion current due to electrons given by:

$$I_n = q\,A\,D_n \frac{\partial N}{\partial x} \qquad Eq. 1.65$$

And, the diffusion current due to holes is given by:

$$I_p = -q\,A\,D_p\,\frac{\partial P}{\partial x} \qquad Eq.1.66$$

Where D_n , D_p are the diffusion constants for electrons and holes respectively

The negative sign in (Eq.1.66) indicates that the hole current flows in the direction opposite to the gradient of the holes. The diffusion constants related with the mobility by Einstein relationship as:

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = V_T = \frac{KT}{q} = {}^{T}\!/_{11600} \qquad Eq.1.67$$

For lightly doped Si at room temperature μ = 39D. The total current of electrons and holes components given by the summing of its diffusion and drift currents as:

$$I_n = qA\left(n\,\mu_n\,E + D_n\,{}^{\partial n}\!/_{\partial x}\right) \qquad Eq.1.68$$

$$I_p = qA\left(p\,\mu_p\,E + D_p\,{}^{\partial n}\!/_{\partial x}\right) \qquad Eq.1.69$$

Solved Examples

Example (43)

An abrupt silicon p-n junction (N_a = 10^{16} cm^{-3} and N_d = 4 x 10^{16} cm^{-3}) is biased with V_a = 0.6 V. Calculate the ideal diode current assuming that the n-type region is much smaller than the diffusion length with W_n = 1 µm and assuming a "long" p-type region. Use μ_n = 1000 cm²/V-s and μ_p = 300 cm²/V-s. The minority carrier lifetime is 10 µs and the diode area is 100 µm by 100 µm.

Solution:

The current calculated from:

$$I = q\,A\left[\frac{D_n n_{po}}{L_n} + \frac{D_p p_{no}}{W_n}\right]\left(e^{V_o/V_t} - 1\right)$$

With

$D_n = \mu_n V_t$ = 1000 x 0.0258 = 25.8 cm²/V-s

$D_P = \mu_P V_t$ = 300 x 0.0258 = 7.75 cm²/V-s

$n_{p0} = n^2_i / N_a = 10^{20}/10^{16} = 10^4$ cm^{-3}

$p_{n0} = n^2_i / N_d = 10^{20}/4 \times 10^{16} = 2.5 \times 10^3$ cm^{-3}

$$L_n = \sqrt{D_n \tau_n} = \sqrt{25.8\,x\,10^{-5}} = 161\,\mu m$$

Yielding I = 40.7 µA

Note that the hole diffusion current occurs in the "short" n-type region and therefore depends on the quasi-neutral width in that region. The electron diffusion current occurs in the "long" p-type region and therefore depends on the electron diffusion length in that region.

Example (44)

For a *p*⁺-*n* Si junction the reverse current at room temperature is 0.9 nA/cm². Calculate the minority-carrier lifetime if N_d = 10^{15} cm^{-3}, n_i = 1.05 × 10^{10} cm^{-3}, and μ_p = 450 $cm^2\,V^{-1}\,s^{-1}$.

Solution:

For a *p*⁺-*n* junction

$$J_S = \frac{e\,p\,D_p}{L_p} = \frac{e\,n_i^2\,D_p}{N_d L_p} = \frac{e\,n_i^2}{N_d}\left(\frac{D_p}{\tau_p}\right)^{1/2}$$

Taking into account that µ = eD/kT, we finally get τ_p = 4.5 × 10^{-9} s.

1.13. LIFE TIME OF CHARGE CARRIERS

In intrinsic material n = p, and due to thermal excitation new hole - electron pairs are produced and others are disappeared as a result of recombination after a time τ_n , τ_p (mean lifetime). The lifetime of carriers is in the range from nanosecond to some hundreds of microseconds. **Consider a Si bar of N-type illuminated by light of the proper frequency, as a result, n and p concentration will increase by the same amount, so:**

$$p_{n0}^{-} - p_{n0} = n_{n0}^{-} - n_{no} \qquad Eq. 1.70$$

Where p_{no}, n_{no} are the equilibrium concentration of holes and electrons, and p_{no}^{-}, n_{no}^{-} represents the carrier concentration during the process. If the source of light is turned off, the carrier concentration will return to its equilibrium values exponentially and with time constant $\tau = \tau_n = \tau_p$, and we can write:

$$p_n - p_{n0} = (p_{n0} - p_{n0}^{-})\, e^{-t/\tau} \qquad Eq.1.71$$

$$n_n - n_{n0} = (n_{n0} - n_{n0}^{-})\, e^{-t/\tau} \qquad Eq.1.72$$

From the equations above, the rate of concentration change for hole is:

$$\frac{\partial\, p_n}{\partial\, t} = -\frac{p_n - p_{n0}}{\tau} = \frac{\partial (p_n - p_{n0})}{\partial\, t} \qquad Eq.1.73$$

And, the rate of concentration change for electrons is:

$$\frac{\partial\, n_n}{\partial\, t} = -\frac{n_n - n_{n0}}{\tau} = \frac{\partial (n_n - n_{n0})}{\partial\, t} \qquad Eq.1.74$$

The quantity p_n - p_{no} or n_n - n_{no} represents the excess carrier density, and the rate of change of excess density is proportional to the density itself. The (-) sign indicates that the change is decreases in case of recombination.

1-14 CONTINUITY EQUATIONS

carrier concentration is a function of both time and distance, upon fact that charge created or destroyed. Consider a volume of area A and length dx, fig.1.29, in which the hole concentration is P, and P/τ_p is the decreasing of hole concentration by recombination, so, the change in hole

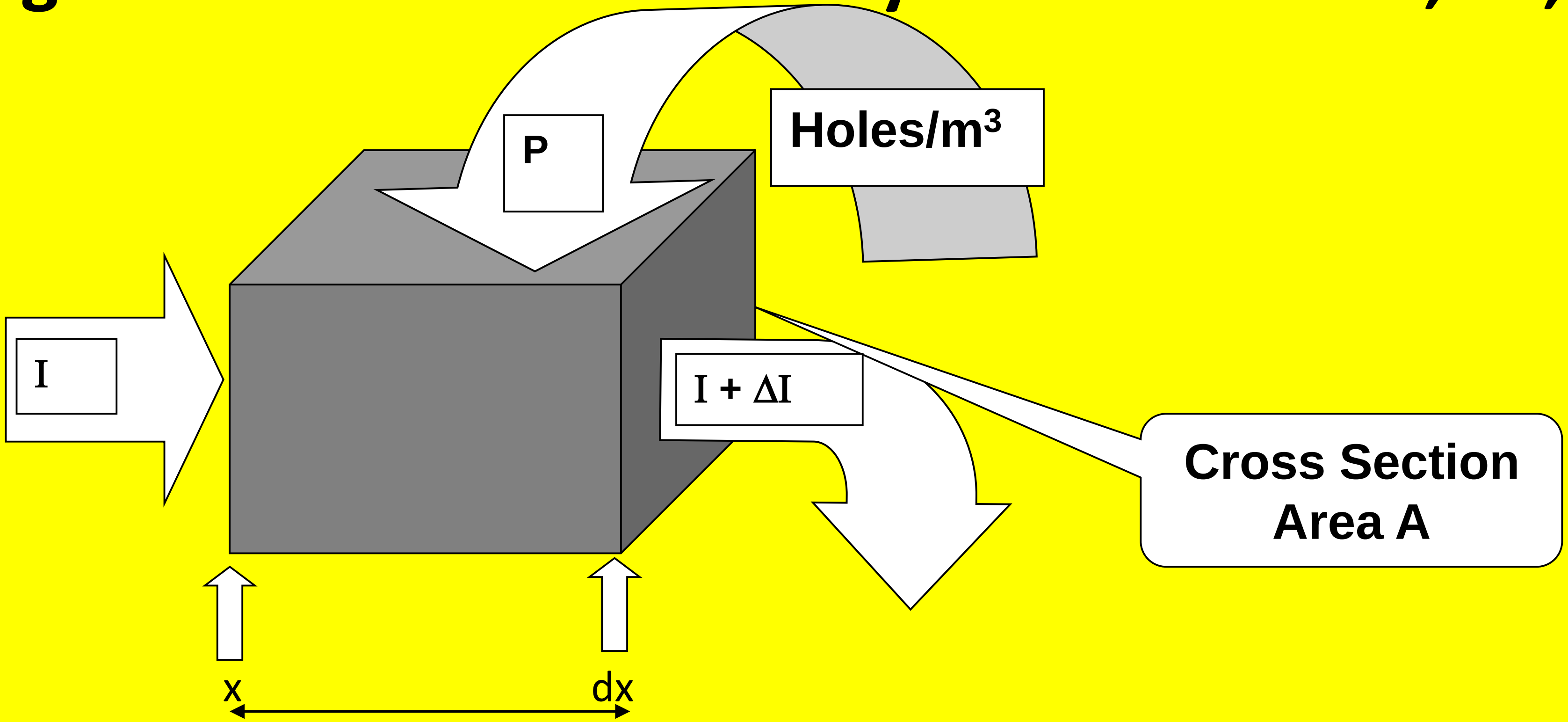


Figure 1.29 Carriers concentration in volume

concentration by recombination within volume (Adx) is:

$$q\,A\,\partial x\ {}^{p}\!/_{\tau_p} \qquad Eq.1.75$$

And if g is the thermal rate for generation hole - electron pair/unit volume, then the increase of concentration by generation is,

$$q\,A\,\partial x\ g \qquad Eq.1.76$$

In general, current vary with distance, current entering volume at x is I and leaving at x+dx is I+dI, then change in current within volume is dI. Let the increase in current due to diffusion within volume is q A dx ∂p/∂t , since . charge tends to equilibrium, then,

$$q\,A\,\partial x\ {}^{\partial p}\!/_{\partial_t} = -\,q\,A\,\partial x\ {}^{p}\!/_{\tau_p} + q\,A\,g\,\partial x - \partial I \qquad Eq.1.77$$

In addition, since the hole- current is the sum of the diffusion current and the drift current components, as:

$$I_p = -\, q\, A\, D_p\, {}^{\partial p}/_{\partial x} + q\, A\, p\;\; \mu_p\, \varepsilon \qquad Eq.1\,.78$$

If the semiconductor is in thermal equilibrium and no electric field is biased, then the hole density will attain a constant value p_o, under this condition, I = 0 and ∂p/∂t = 0, ε = 0, so that:

$$g = {}^{p_0}/_{\tau_p} \qquad Eq.1.79$$

Equation 1.79 indicates those rate at which holes generated thermally equal the rate of recombination under equilibrium condition. Combining equations 77,78,79, we obtain the continuity equation or equation of conservation of charges:

$$\frac{\partial p}{\partial t} = -\, \frac{p -\; p_0}{\tau_p} + \; D_p\, \frac{\partial^2\, p}{\partial\, x^2} - \; \mu_p\, \frac{\partial\, (p\; \varepsilon)}{\partial x} \qquad Eq.1.80$$

If we consider holes in n-type, Eq.1.80 becomes:

$$\frac{\partial p_n}{\partial t} = -\, \frac{p_n - p_{n0}}{\tau_p} + \; D_p\, \frac{\partial^2\, p_n}{\partial\, x^2} - \; \mu_p\, \frac{\partial\, (p_n\; \varepsilon)}{\partial x} \qquad Eq.1.81$$

The general continuity equation 1.81 be considered for three special cases.

1.<u>When concentration is independent of x at zero bias:</u>

If ε = 0 and concentration is independent of x, Eq.1.81 will be rewrite as:

$$\frac{\partial p_n}{\partial t} = -\frac{p_n - p_{n0}}{\tau_p} \qquad Eq. 1.82$$

The solution of (Eq.1.82) is:

$$p_n - p_{n0} = (\bar{p}_{n0} - p_{no})\ e^{-t/\tau} \qquad Eq. 1.83$$

2.When concentration is independent of t at zero bias:

If ε = 0, and a steady state has been reached at no time, so that $\partial p_n/\partial t = 0$, Eq.1.81 will be rewritten again as:

$$D_p \frac{\partial^2 p_n}{\partial x^2} = - \frac{p_n - p_{n0}}{\tau_p} = \frac{\partial^2 p_n}{\partial x^2} = \frac{p_n - p_{n0}}{D_p \tau_p} \qquad Eq.1.84$$

The solution of Eq.1.84 is:

$$p_n - p_{n0} = K_1 e^{-x/L_p} + K_2 e^{x/L_p} \qquad Eq.1.85$$

Where K_1, K_2 are integration constants, $L_p = \sqrt{D_p \tau_p}$ and it represents the distance into the semiconductor at which the injected concentration falls to 1/e of its value at distance x = 0.

3 . When concentration varies sinusoidal with t at zero bias :

If ε = 0, and the injected concentration varies sinusoidal with angular frequency (ω),

$$p_n\,(x\,,t) = \; p_n\,(x)\,e^{j\,\omega\,t} \qquad Eq. \quad 1.86$$

If Eq.1.86 substituted in to the continuity Eq.1.81, results:

$$j\,\omega\,p_n\,(x) = \; -\,\frac{p_n\,(x)}{\tau_p} + \; D_p\,\frac{\partial^2 p_n}{\partial\,x^2} \qquad Eq.1.87$$

$$\frac{\partial^2 p_n}{\partial\,x^2} = \; \frac{1 + j\,\omega\,\tau_p}{L_p^2}\;p_n \qquad Eq.1.88$$

And at F = 0, Eq.1.88 rewritten again as:

$$\frac{\partial^2 p_n}{\partial\,x^2} = \frac{p_n}{L_p^2} \qquad Eq.1.89$$

Solved Example

Example (48)

Consider n-type silicon with $N_d = 10^{15}\ cm^{-3}$ at T = 300◦K. light source is turned on at t = 0. The source illuminates the semiconductor uniformly, generating carriers at the rate of $G_n = G_p = 10^{19} cm^{-3}s^{-1}$. There is no applied field.

(a) Write down the continuity equation and solve it to get the expression for the excess minority carrier concentration, $\delta p(t)$, as a function of time for t ≥ 0.

Solution:

When there is no applied electric field, the carrier distribution is diffusion driven. The continuity equation for the minority carrier

$$\frac{\partial p(x\,,t)}{\partial t} = \frac{1}{q}\frac{\partial\, J_p\,(x\,,t)}{\partial x} + G_p(x\,,t) - R_p(x\,,t)$$

Then reduces to ,

$$\frac{\partial \delta_p}{\partial t} = G_p - \frac{\partial \delta_p}{\tau_p}$$

With the general solution

$$\delta_p(t) = A\, exp\left(-\frac{t}{\pi_p}\right) + G_p\tau_p$$

Using the initial condition (before the light was turned on) that $\delta p(t)$ = 0, then we found that $A = -G_p\tau$. The full solution then is,

$$\delta_p(t) = G_p\tau_p\left\{1 - exp\left(-\frac{t}{\tau_p}\right)\right\}$$

(b) As t >∞, the system will approach steady state. When the steady state excess carrier concentration is $5{\times}10^{13} cm^{-3}$, get the minority carrier lifetime, τ_p.

Solution:

The system will approach steady state as t >∞, Evidently, the steady state carrier density is given

$$\delta p(t)|t \rightarrow \infty\ = G_p\tau_p.$$

The τ_p must then take on the value

$$\tau_p = {}^{5\,x\,10^{13}}/_{G_p}\,.$$

With $G_p = 10^{19}\ cm^{-3}s^{-1}$, then the minority (hole) carried lifetime must be $\tau_p = 5 \times 10^{-6}$ s.

(c) Determine the time at which the excess carrier concentration becomes half of the steady state value, $\delta p(t)|t \rightarrow \infty$ that you calculated in (b).

Solution:

The value at which

$$exp\left(-\frac{t}{\tau}\right) = \frac{1}{2}\ \ so, t = ln\,(2)\tau_p$$
$$= 0.69\ x\ 5\ x\ 10^{-6}\ s$$

1-15. Flux of Charged Spices

To determine the formula of flux (F), $F = -D\,\frac{\partial N}{\partial x}$ consider the motion of positively charged impurities in a crystal. The atom of the crystal forms a series of potential hills, which hinder the motion of the charged impurities as shown. The height of potential barrier (W) is in the order of some of electron volt. In most material, the distance between successive potential barrier (a) is in the order of the lattice spacing which is in some angstroms (1 °A = 10^{-10} meter).

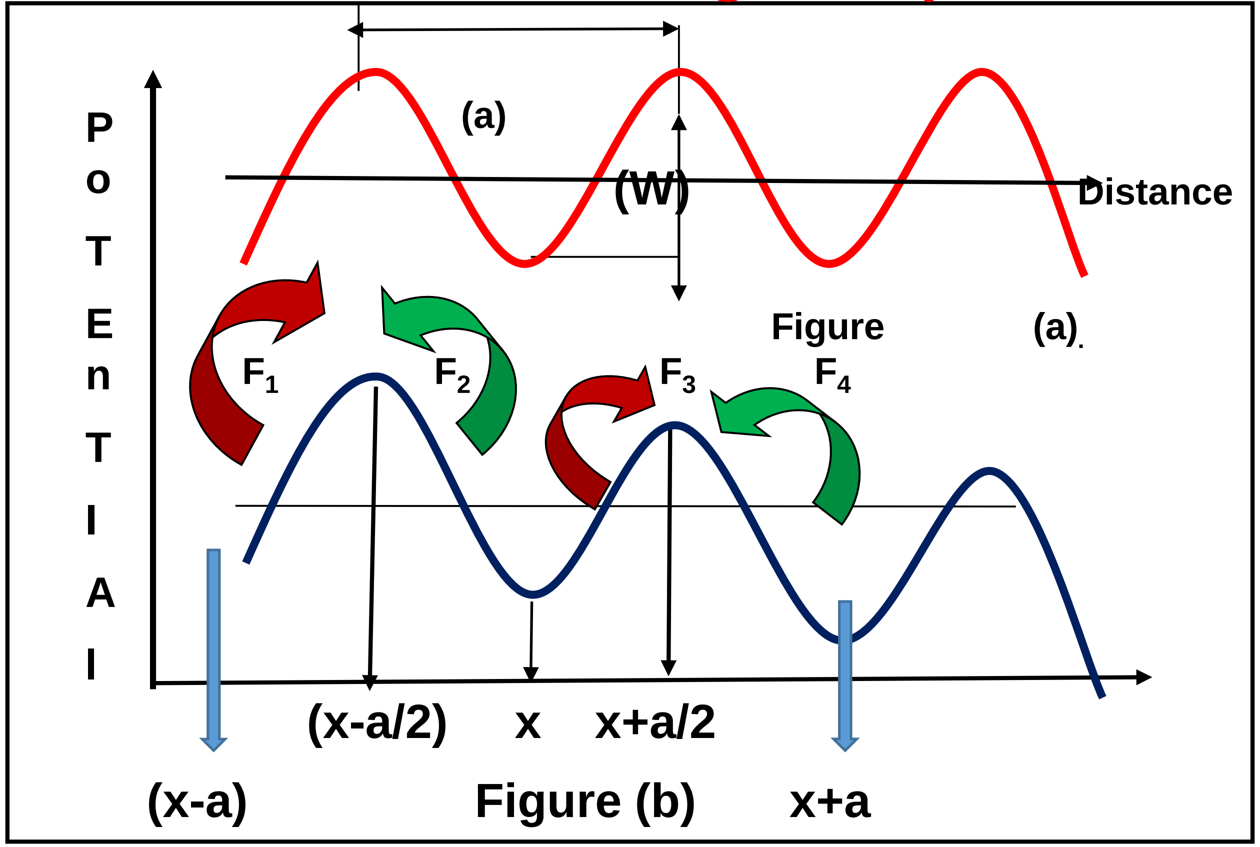


Fig. (1 - 30) Model of ionic motion within crystal, Potential distribution with and without bias

If a constant field is applied, the potential distribution as a function of distance is shown in figure b, this make flow of positively charged particles to the right easier than left. To calculate the flux (F) at a position (x), it is average of fluxes at positions (x – a/2) and at (x + a/2), these two fluxes given by ($f_1 - f_2$), and ($f_3 - f_4$).Consider component (F_1), is product of:

1. Intensity per unit area for impurities (charges) at the potential valley at (x – a).
2. The probability of a jump of any of these impurities (charges) to the next valley at position (x)
3. The frequency of attempted jump (ν)

Thus, we can write:

$$F_1 = [a\,C\,(x-a)]\,exp - \frac{q}{K\,T}\left(W - \frac{1}{2}\;a\,\epsilon\right)(\nu)\;\mathbf{Eq}\;(1-90)$$

Where, $[a\,C\,(x-a)]$ is the density per unit area of the particles situated in the valley at (x – a). The exponential factor is the probability of a jump from the valley at (x – a) to the valley at position (x), and (ν) is frequency of attempted jump note that, the lowering of the barrier due to the electric field (ε).

Similar formulas can be written for (F_2), (F_3), and (F_4). By combined them to give a formula for the flux (F) at position (x), with the concentration (C(x ± a)) approximated by

$$(\,\mathrm{C}(\mathrm{x}) \;\pm\; \mathrm{a}\,({}^{\partial\,\mathrm{C}}/{}_{\partial\,\mathrm{x}}),$$ We obtain,

$$F(x) = -\left(\mathsf{v}\, a^2\, e^{-qW/KT}\right)\frac{\partial C}{\partial x}\cosh\frac{q\, a\, \varepsilon}{2\, KT} + \left(2\, a\, \mathsf{v}\, e^{-qW/Kt}\right)C \sinh\frac{q\, a\, \varepsilon}{2\, KT}\quad Eq\ (1-91)$$

As extremely important, limiting from of this equation is obtained for the case when the electric field is relatively small, i.e., ε «KT/q a. In this case, we can expand the cosh and the sinh terms in the equation. Noting that cosh (x) = 1 and sinh (x) = x for x → 0, this results in the limiting form of the flux equation for a positively charged spices.

$$F\,(x) = -D\,\frac{\partial\, C}{\partial\, x} + \mu\,\varepsilon\, C \quad Eq.(1-92)$$ **, Where:**

$$D = v\mathrm{a}^2 e^{-qW/KT}$$

$$\mu = \frac{v\mathrm{a}^2 e^{-qW/KT}}{KT/q}$$

Note that, mobility (μ) and the diffusivity (D) related by:

$$D = \frac{KT}{q}\mu$$

This is the well Known Einstein relationship. A similar derivation made for the motion of negatively charged spices. In addition, we can derive that,

$$D = D_o e^{-E_a/KT}$$

Where, (D_o) is the diffusivity of impurities in oxide, (E_a) is the activation energy. Thus, the activation energy corresponds to the energy required to form a silicon space rather than to the energy required to move the impurity. Since silicon – to – silicon, bunds must be broken to form a space. In fact, activation energy of the diffusivity of acceptor and donor type impurities, and it is in the range of two and three electron volt in germanium.