Solved Questions
Semiconductor Principle

Questions (1)

Don't just sit there! Build something!!

Learning to mathematically analyze circuits requires much study and practice. Typically, students practice by working through lots of sample problems and checking their answers against those provided by the textbook or the instructor. While this is good, there is a much better way. You will learn much more by building *and analyzing real circuits*, letting your test equipment provide the "answers" instead of a book or another person. For successful circuit-building exercises, follow these steps:

1. **Carefully measure and record all component values prior to circuit construction, choosing resistor values high enough to make damage to any active components unlikely.**
2. **Draw the schematic diagram for the circuit to be analyzed.**
3. **Carefully build this circuit on a breadboard or other convenient medium.**
4. **Check the accuracy of the circuit's construction, following each wire to each connection point, and verifying these elements one-by-one on the diagram.**
5. **Mathematically analyze the circuit, solving for all voltage and current values.**
6. **Carefully measure all voltages and currents, to verify the accuracy of your analysis.**
7. **If there are any substantial errors (greater than a few percent), carefully check your circuit's construction against the diagram, then re-calculate the values and re-measure.**

When students are first learning about semiconductor devices, and are most likely to damage them by making improper connections in

their circuits, I recommend they experiment with large, high-wattage components (1N4001 rectifying diodes, TO-220 or TO-3 case power transistors, etc.), and using dry-cell battery power sources rather than a benchtop power supply. This decreases the likelihood of component damage.

As usual, avoid very high and very low resistor values, to avoid measurement errors caused by meter "loading" (on the high end) and to avoid transistor burnout (on the low end). I recommend resistors between 1 kΩ and 100 kΩ.

One way you can save time and reduce the possibility of error is to begin with a very simple circuit and incrementally add components to increase its complexity after each analysis, rather than building a whole new circuit for each practice problem. Another time-saving technique is to re-use the same components in a variety of different circuit configurations. This way, you won't have to measure any component's value more than once.

Solution

Let the electrons themselves give you the answers to your own "practice problems"!

Notes:

It has been my experience that students require much practice with circuit analysis to become proficient. To this end, instructors usually provide their students with lots of practice problems to work through, and provide answers for students to check their work against. While this approach makes students proficient in circuit theory, it fails to fully educate them.

Students don't just need mathematical practice. They also need real, hands-on practice building circuits and using test equipment. So, I suggest the following alternative approach: students should *build* their own "practice problems" with real components, and try to mathematically predict the various voltage and current

values. This way, the mathematical theory "comes alive," and students gain practical proficiency they wouldn't gain merely by solving equations.

Another reason for following this method of practice is to teach students *scientific method*: the process of testing a hypothesis (in this case, mathematical predictions) by performing a real experiment. Students will also develop real troubleshooting skills as they occasionally make circuit construction errors.

Spend a few moments of time with your class to review some of the "rules" for building circuits before they begin. Discuss these issues with your students in the same Socratic manner you would normally discuss the worksheet questions, rather than simply telling them what they should and should not do. I never cease to be amazed at how poorly students grasp instructions when presented in a typical lecture (instructor monologue) format!

A note to those instructors who may complain about the "wasted" time required to have students build real circuits instead of just mathematically analyzing theoretical circuits:

What is the purpose of students taking your course?

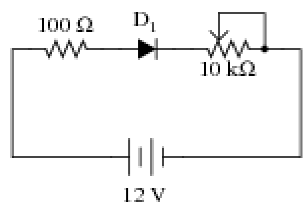
If your students will be working with real circuits, then they should learn on real circuits whenever possible. If your goal is to educate theoretical physicists, then stick with abstract analysis, but most of us plan for our students to do something in the real world with the education we give them. The "wasted" time spent building real circuits will pay huge dividends when it comes time for them to apply their knowledge to practical problems.

Furthermore, having students build their own practice problems teaches them how to perform *primary research*, thus empowering them to continue their electrical / electronic education autonomously.

In most sciences, realistic experiments are much more difficult and expensive to set up than electrical circuits. Nuclear physics, biology, geology, and chemistry professors would just love to be able to have their students apply advanced mathematics to real experiments posing no safety hazard and costing less than a textbook. They can't, but you can. Exploit the convenience inherent to your science, and *get those students of yours practicing their math on lots of real circuits!*

Questions (2)

A student sets up a circuit that looks like this, to gather data for characterizing a diode:



Measuring diode voltage and diode current in this circuit, the student generates the following table of data:

V_{diode}	I_{diode}
0.600 V	1.68 mA
0.625 V	2.88 mA
0.650 V	5.00 mA
0.675 V	8.68 mA
0.700 V	14.75 mA
0.725 V	27.25 mA
0.750 V	48.2 mA

This student knows that the behavior of a PN junction follows Shockley's diode equation, and that the equation may be simplified to the following form:

Where, K = a constant incorporating both the thermal voltage and the nonideality coefficient. The goal of this experiment is to calculate K and I_S, so that the diode's current may be predicted for any arbitrary value of voltage drop. However, the equation must be simplified a bit before the student can proceed. At substantial levels of current, the exponential term is very much larger than unity

$$\left(e^{\{V_{diode}/K\}} \gg 1\right)$$

so the equation may be simplified as such:

$$I_{diode} \approx I_S \left(e^{\{V_{diode}/K\}}\right)$$

From this equation, determine how the student would calculate K and I_S from the data shown in the table. Also, explain how this student may verify the accuracy of these calculated values.

Solution

$K \approx 0.04516$, $I_S \approx 2.869$ nA

Hint: this may be a difficult problem to solve if you are unfamiliar with the algebraic technique of dividing one equation by another. Here is the technique shown in general terms:

Given: $y_1 = ax_1$ $y_2 = ax_2$

From here, it may be possible to perform simplifications impossible before. I suggest using this technique to solve for K first.

Follow-up question: explain how this student knew it was "safe" to simplify the Shockley diode equation by eliminating the "- 1" term. Is this sort of elimination always permissible? Why or why not?

Notes:

The algebraic technique used to solve for K is very useful for certain types of problems.

Discuss the follow-up question with your students. It is important in the realm of technical mathematics to have a good sense of the relative

values of equation terms, so that one may "safely" eliminate terms as a simplifying technique without incurring significant errors. In the Shockley diode equation, it is easy to show that the exponential term is *enormous* compared to 1 for the values of V_{diode} shown in the table (assuming a typical value for thermal voltage), and so the "- 1" part is very safe to eliminate.

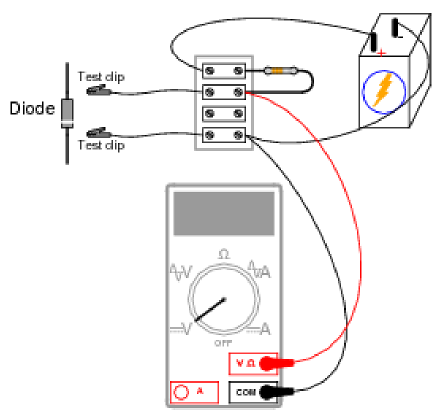
Also, discuss the idea of verifying the calculated values of K and I_S with your students, to help them cultivate a scientifically critical point of view in their study of electronics.

Incidentally, the data in this table came from a real experiment, set up exactly as shown by the schematic diagram in the question. Care was taken to avoid diode heating by turning the potentiometer to maximum resistance between readings.

Questions (3)

Most introductory textbooks will tell you that a silicon PN junction drops 0.7 volts when forward-biased, and a germanium PN junction drops 0.3 volts when forward biased. Design a circuit that tests the "forward voltage" (V_F) of a PN-junction diode, so you may measure the voltage yourself, without the use of a special diode-testing meter.

Solution



Notes:

Ask your students how they would determine the size of resistor to use for this "diode test" circuit. Would be permissible to use any arbitrary value of resistor, or does the value matter significantly?

Questions (4)

Explain the difference between extrinsic and intrinsic semiconductors.

Solution

An extrinsic semiconductor is a semiconductor which contains foreign elements capable of contributing mobile charge carriers, electrons, to the conduction band (n– type) or holes to the valence band (p–type). An intrinsic semiconductor contains no foreign elements.

Questions (5)

(a) How do you expect the conductivity to vary in an intrinsic semiconductor with increasing temperature? Explain your answer.
(b) How do you expect the conductivity to vary in a metallic conductor with increasing temperature?

Solution

(a) From our limited knowledge of the conduction behavior, we must assume that in semiconductors the conductivity will increase with T since the number of electrons in the conduction band increases.
(b) As a first approximation, we can say that the number of free electrons is independent of T; that, however, the atoms in the lattice will, with increasing temperature, be subject to increased oscillations about their (relatively) fixed position. These oscillations provide greater opportunity for "scattering" of conducting electrons and thus will "reduce" the mobility of the electrons. We can expect the electrical conductivity of metals to decrease with increasing temperature.

Questions (6)

The relationship between voltage and current for a PN junction is described by this equation, sometimes referred to as the "diode equation," or "Shockley's diode equation" after its discoverer:

$$I_D = I_S \left(e^{[(qV_D)/NkT]} - 1\right)$$

Where,

I_D = Current through the PN junction, in amps

I_S = PN junction saturation current, in amps (typically 1 picoamp)

e = Euler's number ≈ 2.718281828

q = Electron unit charge, 1.6×10^{-19} coulombs

V_D = Voltage across the PN junction, in volts

N = Nonideality coefficient, or emission coefficient (typically between 1 and 2)

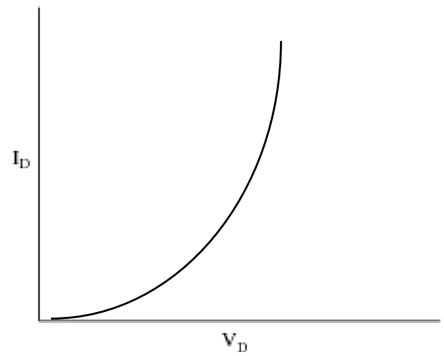
k = Boltzmann's constant, 1.38×10^{-23}

T = Junction temperature, degrees Kelvin

At first this equation may seem very daunting, until you realize that there are only three variables in it: I_D, V_D, and T. All the other terms are constants. Since in most cases we assume temperature is constant as well, we are only dealing with two variables: diode current and diode voltage. Based on this realization, re-write the equation as a proportionality rather than an equality, showing how the two variables of diode current and voltage relate:

$$I_D \propto e^{V_D}$$

Based on this simplified equation, what would an I/V graph for a PN junction look like? How does this graph compare against the I/V graph for a resistor?



Solution

Simplified proportionality:

$$I_D \propto e^{V_D}$$

The graph described by the "diode formula" is a standard exponential curve, rising sharply as the independent variable (V_D, in this case) increases. The corresponding graph for a resistor, of course, is linear.

Notes:

Ask your students to sketch their own renditions of an exponential curve on the whiteboard for all to see. Don't just let them get away with parroting the answer: It's an exponential curve."

Questions (7)

Derive the expression.

$$p = \frac{1}{4}\left(\frac{2\, m_h\, KT}{\pi\, h^2}\right)^{3/2} e^{(E_v - E_F)/KT} = N_v\, e^{(E_v - E_{cF})/KT}$$

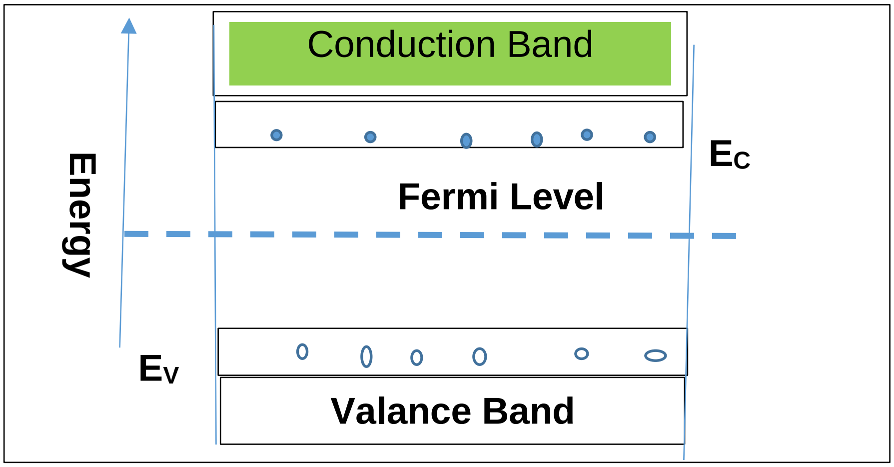
Solution:

For an intrinsic semiconductor, the number of electrons in the conduction band is equal to the number of holes in the valence band since a hole is left in the valence band only when an electron makes a transition to the conduction band,

$$n = p$$

Using this and if the effective masses of the electrons and holes are the same one gets,

$$e^{(E_F - E_c)/kT} = e^{(E_v - E_F)/kT}$$



Giving:

$$E_F = \frac{E_C + E_V}{2} \qquad (C)$$

i.e. the Fermi level lies in the middle of the forbidden gap. Note that there is no contradiction with the fact that no state exists in the gap as is only an energy level and not a state. By substituting the above expression for Fermi energy in(A) or (B),

$$n = \frac{1}{4}\left(\frac{2\, m_e\, KT}{\pi\, h^2}\right)^{3/2} e^{(E_F - E_C)/KT} = N_C\, e^{(E_F - E_C)/KT} \qquad (A)$$

$$p = \frac{1}{4}\left(\frac{2\, m_h\, KT}{\pi\, h^2}\right)^{3/2} e^{(E_v - E_F)/KT} = N_v\, e^{(E_v - E_{cF})/KT} \qquad (B)$$

We obtain an expression for the number density of electrons or holes ($n = p = n_i$)

$$n_i = \frac{1}{4}\left(\frac{2\, KT}{\pi\, h^2}\right)^{3/2} (m_e m_h)^{3/4}\, e^{-\Delta/2\,KT} \qquad (D)$$

Where Δ is the width of the gap

Questions (8)

When "P" and "N" type semiconductor pieces are brought into close contact, free electrons from the "N" piece will rush over to fill holes

in the "P" piece, creating a zone on both sides of the contact region devoid of charge carriers. What is this zone called, and what are its electrical characteristics?

Solution

This is called the *depletion region*, and it is essentially an insulator at room temperatures.

Notes:

Students should know that both "N" and "P" type semiconductors are electrically conductive. So, when a depletion region forms in the contact zone between two differing semiconductor types, the conductivity from end-to-end must be affected. Ask your students what this effect is, and what factors may influence it.

Questions (9)

What happens to the thickness of the depletion region in a PN junction when an external voltage is applied to it?

Solution

The answer to this question depends entirely on the polarity of the applied voltage! One polarity tends to expand the depletion region, while the opposite polarity tends to compress it. I'll let you determine which polarity performs which action, based on your research.

Notes: Ask your students what effect this change in depletion layer thickness has on overall conductivity through the PN junction. Under what conditions will the conductivity be greatest, and under what conditions will the conductivity be least?

Questions (10)

Which are the most important breakdown mechanisms in a reverse-biased p-n junction? Describe succinctly (no more than one paragraph for each process) how they work and how they trigger the breakdown process.

Solution: **The important breakdown mechanisms are Zener and Avalanche.**

Questions (11)

If a semiconductor PN junction is reverse-biased, ideally no continuous current will go through it. However, in real there will be a small amount of reverse-bias current that goes through the junction. How is this possible? What allows this reverse current to flow?

Solution

Minority carriers allow reverse current through a PN junction.

Notes:

Review with your students what "minority carriers" are, and apply this concept to the PN junction. Trace the motions of these minority carriers, and compare them with the motions of majority carriers in a forward-biased PN junction.

Questions (12)

Shockley's diode equation in standard form is quite lengthy, but it may be considerably simplified for conditions of room temperature. Note that if the temperature (T) is assumed to be room temperature (25° C), there are three constants in the equation that are the same for all PN junctions: T, k, and q.

$$I_D = I_S\left(e^{\{qV_D/NKT\}} - 1\right)$$

The quantity [kT/q] is known as the *thermal voltage* of the junction. Calculate the value of this thermal voltage, given a room temperature of 25° C. Then, substitute this quantity into the original "diode formula" to simplify its appearance.

Solution

If you obtained an answer of 2.16 mV for the "thermal voltage," you have the temperature figure in the wrong units!

$$I_D = I_S\left(e^{\{V_D/0.026\,N\}} - 1\right)$$

Notes:

Of course, students must research the difference between degrees Kelvin and degrees Celsius to successfully calculate the thermal voltage for the junction. They will also have to figure out how to substitute this figure in place of q, k, and T in the original equation. The latter step will be difficult for students not strong in algebra skills. For those students, I would suggest posing the following question to get them thinking properly about algebraic substitution. Suppose we had the formula $y = x^{[ab/cd]}$, and we knew that [b/c] could be written as m. How would we substitute m into the original equation? Answer: $y = x^{[am/d]}$.

Questions (13)

To simplify analysis of circuits containing PN junctions, a "standard" forward voltage drop is assumed for any conducting junction, the exact figure depending on the type of semiconductor material the junction is made of.

How much voltage is assumed to be dropped across a conducting *silicon* PN junction? How much voltage is assumed for a forward-biased *germanium* PN junction? Identify some factors that cause the real forward voltage drop of a PN junction to deviate from its "standard" figure.

Solution

Silicon = 0.7 volts; Germanium = 0.3 volts.

Temperature, current, and doping concentration all affect the forward voltage drop of a PN junction.

Notes: I've seen too many students gain the false impression that silicon PN junctions *always* drop 0.7 volts, no matter what the conditions. This "fact" is emphasized so strongly in many textbooks that students usually don't think to ask when they measure a diode's

forward voltage drop and find it to be considerably different than 0.7 volts! It is very important that students realize this figure is an approximation only, used for the sake of (greatly) simplifying junction semiconductor circuit analysis.

Questions (14)

The nonconducting depletion region of a PN junction forms a parasitic capacitance between the P and the N semiconductor region. Does the capacitance increase or decrease as a greater reverse-bias voltage is applied to the PN junction? Explain your answer.

Solution

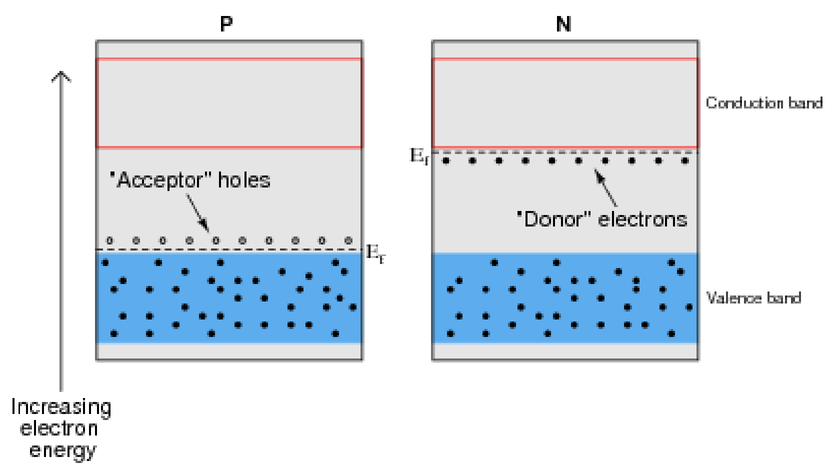
The junction capacitance will decrease as the reverse-bias voltage across the junction increases. Challenge question: can you think of any practical applications for this variable-capacitance effect?

Notes:

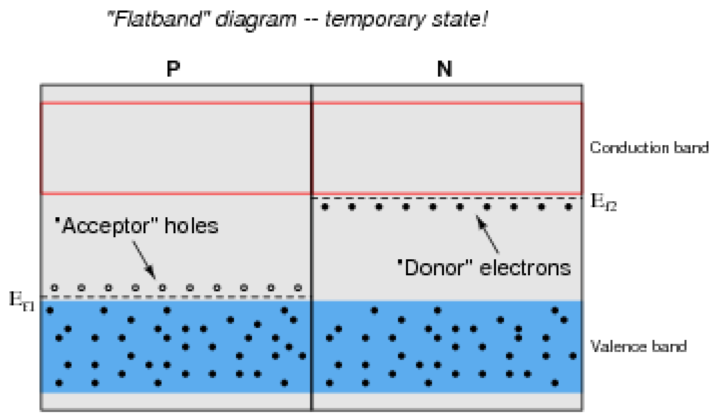
This question is a good review of capacitor theory, and also an opportunity to introduce a special kind of diode: the *varactor*.

Questions (15)

Shown here are two energy diagrams: one for a "P" type semiconducting material and another for an "N" type.



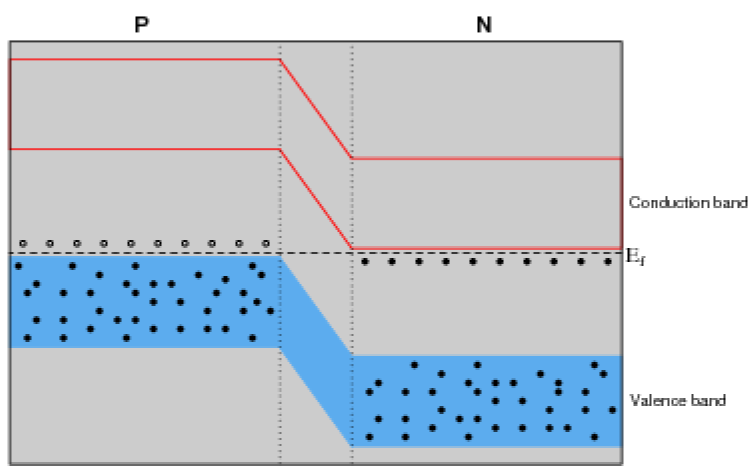
Next is an energy diagram showing the *initial* state when these two pieces of semiconducting material are brought into contact with each other. This is known as a ***flat band diagram***:



The state represented by the "flat band" diagram is most definitely a temporary one. The two different Fermi levels are incompatible with one another in the absence of an external electric field. Draw a new energy diagram representing the final energy states after the two Fermi levels have equalized.

Note: E_f represents the Fermi energy level, and not a voltage. In physics, E always stands for energy and V for electric potential (voltage).

Solution



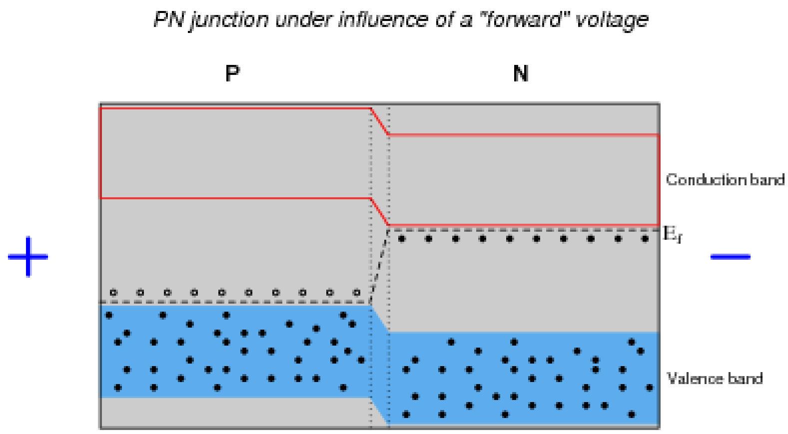
Electrons from the N-piece rushed over to fill holes in the P-piece *to achieve a lower energy state* and equalize the two Fermi levels. This displacement of charge carriers created an electric field which accounts for the sloped energy bands in the middle region.

Follow-up question: what is this middle region called?

Notes:
This is one of those concepts I just couldn't understand when I had no comprehension of the quantum nature of electrons. In the "planetary" atomic model, there is no reason whatsoever for electrons to move from the N-piece to the P-piece unless there was an electric field pushing them in that direction. And conversely, once an electric field was created by the imbalance of electrons, the free-wheeling planetary theory would have predicted that the electrons move right back where they came from to neutralize the field.
Once you grasp the significance of quantized energy states, and the principle that particles do not "hold on" to unnecessary energy and therefore remain in high states when they could move down to a lower level, the concept becomes much clearer.

Questions (16)
Draw an energy diagram for a PN semiconductor junction under the influence of a *forward* external voltage.

Solution



Note: E_f represents the Fermi energy level, and not a voltage. In physics, E always stands for energy and V for potential (voltage).

Notes:

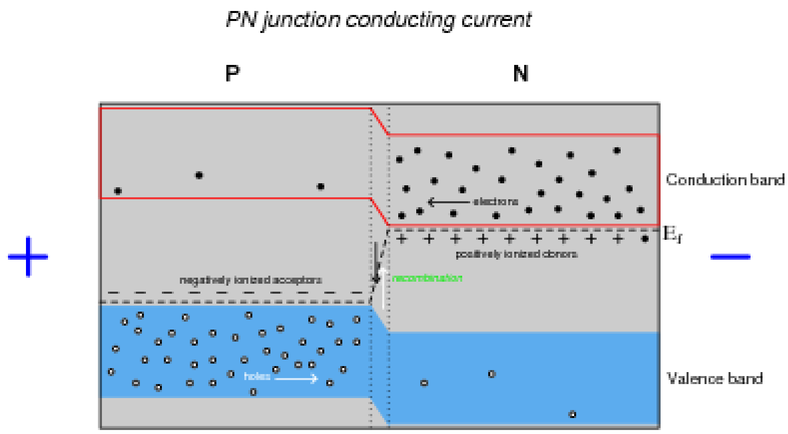
Here it is very important that students understand the effects an electric field has on energy bands.

Questions (17)

Draw an energy diagram for a PN semiconductor junction showing the motion of electrons and holes conducting an electric current.

Solution

For the sake of clearly seeing the actions of charge carriers (mobile electrons and holes), non-moving electrons in the valence bands are not shown:

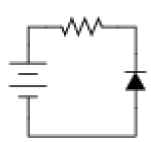


The "+" and "-" signs show the locations of ionized acceptor and donor atoms, having taken on electric charges to create valence-band holes and conduction-band electrons, respectively.

Note: E_f represents the Fermi energy level, and not a voltage. In physics, E always stands for energy and V for potential (voltage).

Notes:
Students will probably ask why there are a few holes shown in the N-type valence band, and why there are a few electrons in the P-type conduction band. Let them know that just because N-type materials are specifically designed to have conduction-band electrons does not mean they are completely devoid of valence-band holes, and visa-versa! What your students see here are *minority carriers*.

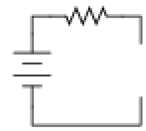
Questions (18)
Is this diode *forward-biased* or *reverse biased*?



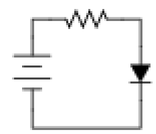
Solution: This diode is reverse-biased.

Notes:
Nothing much to comment on here!

Questions (19)
Insert a diode into this circuit schematic in the correct direction to make it *forward-biased* by the battery voltage:



Solution



Notes:
Nothing much to comment on here!

Questions (20)

What you know about p-n diode with a "long" quasi-neutral region

Solution
Only decaying exponential terms yield a finite carrier density far away from the depletion region so that the coefficients *B* and *C* must be zero, yielding:

$$p_n(x \geq x_n) = p_{n0} + p_{n0}\left(e^{V_a/V_t} - 1\right)e^{-(x-x_n)/L_p} \quad \textbf{(pnc5)}$$

and

$$n_p(x \leq -x_p) = n_{p0} + n_{p0}\left(e^{V_a/V_t} - 1\right)e^{(x+x_p)/L_p} \quad \textbf{(pnc6)}$$

were the constants *A* and *D* were chosen so that the boundary conditions at $x = x_n$ and $x = -x_p$ are satisfied.

The diffusion current densities in the quasi-neutral regions due to minority carriers is then obtained from the derivative of the minority carrier density, yielding:

$$J_p(x \geq x_n) = -qD_p \frac{dp_n}{dx} = q\frac{D_p\, p_{n0}}{L_p}(e^{V_a/V_t} - 1)e^{-\frac{(x-x_n)}{L_p}}$$

(pnc7)

$$J_n(x \leq -x_p) = -qD_n \frac{dn_p}{dx} = q\frac{D_n\, n_{p0}}{L_n}(e^{V_a/V_t} - 1)e^{\frac{(x+x_p)}{L_n}}$$

(pnc8)

The total current must be constant throughout the structure since a steady state case is assumed: no charge can accumulate or disappear somewhere in the structure so that the charge flow must be constant throughout the diode. The total current then equals the sum of the maximum electron current in the p-type region, the maximum hole current in the n-type regions and the current due to recombination within the depletion region. The maximum currents in the quasi-neutral regions occur at either side of the depletion region. Also since we do not know the current due to recombination in the depletion region we will simply assume that it can be ignored. Later on we will more closely examine this assumption. The total current is then given by:

$$I = A\,[J_n(x = -x_p) + J_p(x = x_n) + J_r] \cong I_s\,(e^{V_a/V_t} - 1)$$

(pnc9)

where I_s can be written in the following forms:

$$I_s = q\,A\,[\frac{D_n\,n_{p0}}{L_n} + \frac{D_p\,p_{n0}}{L_p}] = q\,A\,[\frac{n_{p0}\,L_n}{\tau_n} + \frac{p_{n0}\,L_p}{\tau_p}]$$

(pnc12)

using the definition of the diffusion length, namely

$$L_n = \sqrt{D_n\,\tau_n}$$ **(pnc14)**

and

$$L_p = \sqrt{D_p\,\tau_p}$$ **(pnc15)**

while the thermal equilibrium minority carrier densities, n_{p0} and p_{n0}, are given by:

$$n_{p0} = \frac{n_i^2}{N_a}$$ **(pnc2a)**

and

$$p_{n0} = \frac{n_i^2}{N_d}$$ **(pnc1a)**

We now come back to our assumption that the current due to recombination in the depletion region can be simply ignored. Given that there is recombination in the quasi-neutral region it would be unreasonable to suggest that the recombination rate would simply drop to zero in the depletion region. Instead we assume that the recombination rate is constant in the depletion region. In the next section we will show that this assumption is correct if band-to-band recombination dominates in the depletion region and that it underestimates the total recombination current if Shockley-Hall-

Read recombination dominates. To further simplify the analysis, we will consider a p^+-n junction so that we only need to consider the recombination in the n-type region. The current due to recombination in the depletion region is therefore given by:

$$I_r \geq q\,A\,\frac{p_{n0}\,x_n}{\tau_p}(e^{V_a/V_t}-1) \quad \textbf{(pnc10)}$$

so that I_r can be ignored if:

$$I_r << I,\ \text{for}\ x_n << L_p \quad \textbf{(pnc11)}$$

A necessary, but not sufficient requirement is therefore that the depletion region width is much smaller than the diffusion length for the ideal diode assumption to be valid. Silicon and germanium p-n diodes almost always satisfy this requirement, while gallium arsenide p-n diodes rarely do because of the short carrier life time and diffusion length.

Questions (21)

What you know about The p-n diode with a "short" quasi-neutral region

Solution

A "short" diode is a diode with quasi-neutral regions which are much shorter than the minority carrier diffusion lengths. The quasi-neutral regions widths, w_n' and w_p', equal the physical widths of the n-type and p-type regions, w_n and w_p, minus the depletion layer widths, x_n and x_n, or:

$$w_n' = w_n - x_n$$

and

$$w_p' = w_p - x_p$$

As the quasi-neutral region is much smaller than the diffusion length

one finds that recombination in the quasi-neutral region is negligible so that the diffusion equation is reduced to:

$$0 = D_n \frac{d^2 n_p}{dx^2}, \text{ and } 0 = D_p \frac{d^2 p_n}{dx^2} \quad \textbf{(pnc13a)}$$

so that the carrier density varies linearly throughout the quasi-neutral region and in general is given by:

$$n_p = A + B\,x \text{ , and } p_n = A + B\,x \quad \textbf{(pnc13b)}$$

where *A* and *B* are constants are obtained by satisfying the boundary conditions. Applying the same boundary conditions at the edge of the depletion region as above and requiring thermal equlibrium at the contacts yields:

$$p_n = p_{n0} + p_{n0}\,(e^{V_a/V_t} - 1)\left(1 - \frac{(x - x_n)}{w_n^{'}}\right) \quad \textbf{(pnc13c)}$$

for the hole density in the n-type quasi-neutral region.

The current in a "short" diode is again obtained by adding the maximum diffusion currents in each of the quasi-neutral regions and ignoring the current due to recombination in the depletion region, yielding:

$$I = A\,[J_n(x = -x_p) + J_p(x = x_n) + J_r] \cong I_s\,(e^{V_a/V_t} - 1)$$

(pnc9)

where the saturation current, I_s is given by:

$$I_s = q\,A\,[\frac{D_n\,n_{p0}}{w_p^{'}} + \frac{D_p\,p_{n0}}{w_n^{'}}] \quad \textbf{(pnc13)}$$

A comparison of the "short" diode expression with the "long" diode

expression reveals that they are the same except for the use of either the diffusion length or the quasi-neutral region width in the denominator, whichever is smaller.