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Part (1) Solved PROBLEMS
Visualization of the Silicon Crystal

Example (1):

Write the electronic configuration of silicon has 14 electrons in its atom, determine in which sub shell and in which orbit and how many electrons in the highest sub shell energy.

Solution:

The electronic configuration of silicon as follows,

$$1S^2\ 2S^2\ 2p^6\ 3S^2\ 3p^2$$

The highest sub shell energy lies in (M) orbit, in the sub shell (p) which is not fully occupied , it has only 2 electrons

Example (2):

Write the electronic configuration of iodine (I) has 53 electrons in its atom, determine in which sub shell and in which orbit and how many electrons in the highest sub shell energy.

Solution:

The electronic configuration of iodine (I) as follows,

$$1S^2\ 2S^2\ 2p^6\ 3S^2\ 3p^6\ 3D^{10}\ 4S^2\ 4p^6\ 4D^{10}\ 4F^7$$

The highest sub shell energy lies in (N) orbit, in the sub shell (F) is not fully occupied, it has only seven electrons

Example (3):

Write the electronic configuration of Tantalum 73 has 73 electrons in its atom, determine in which sub shell and in which orbit and how many electrons in the highest sub shell energy.

Solution:

The electronic configuration of iodine (I) as follows,

$$1S^2\ 2S^2\ 2p^6\ 3S^2\ 3p^6\ 3D^{10}\ 4S^2\ 4p^6\ 4D^{10}\ 4F^{14}\ 5S^2\ 5p^6\ 5D^5$$

The highest sub shell energy lies in (O) orbit, in the sub shell (D) which is not fully occupied, it has only 5 electrons

Example (4) :

Write the electron configuration of silicon in its following energy states $_{14}Si^{+}$. $_{14}Si^{+2}$, $_{14}Si^{+3}$.

Solution:

From tables, the atomic number for silicon is 14, which means, it has 14 electrons in its steady state distributed among orbits and sub shells. Therefore, the electron configuration in the following energy states $_{14}Si^{+}$. $_{14}Si^{+2}$, $_{14}Si^{+3}$ can be represented as,

- **$_{14}Si^{+}$ has 14-1=13 electrons**

  $$1s^2 2s^2 2p^6 3s^2 3p^1$$

- **$_{14}Si^{+2}$ has 14-2=12 electrons**

  $$1s^2 2s^2 2p^6 3s^2$$

- **$_{14}Si^{+3}$ has 14-3=11 electrons**

  $$1s^2 2s^2 2p^6 3s^1$$

Problem 1

The expression for the Bohr radius can also be applied to the hydrogen-like atom consisting of an ionized donor and the electron provided by the donor. Modify the expression for the Bohr radius so that it applies to this hydrogen-like atom. Calculate the resulting radius of an electron orbiting around the ionized donor in silicon. (ϵ_r = 11.9 and μ_e^* = 0.26 μ_0)

Solution

The Bohr radius is obtained from:

$$a_o = \frac{\epsilon_o h^2 n^2}{p\,\mu_0 q^2}$$

However, since the electron travel through silicon one has to replace the permittivity of vacuum with the dielectric constant of silicon and the free electron mass with the effective mass for conductivity calculations so that:

$$a_o, donor\ in\ silicon = a_o \frac{\epsilon_r}{\mu_e^* \mu_0} = 529\ x\ \frac{11.9}{0.26}\ pm = 2.42\ nm$$

Problem 2

Electron mobility in Si is 1400 cm^2 V^{-1}s^{-1}. Calculate the mean free time in scattering, (Relaxationszeit) of electrons. Effective mass is $m^*_e/m_0 = 0.33$.

Solution: (1.3)

From $\mu = e\tau/m^*$ we get that $\tau = 2.6 \times 10^{-13}$ s.

Problem 3

Calculate thermal velocity of electrons and holes in GaAs at room temperature. Effective masses are $m^*_e/m_0 = 0.063$ and $m^*_h/m_0 = 0.53$.

Solution:

$$v_t = \frac{\int_0^\infty v\ exp\left(-\ m^*\ {v^2}/{2\ KT}\right)\ d^3v}{\int_0^\infty\ exp\left(-\ m^*\ {v^2}/{2\ KT}\right)\ d^3v} = \sqrt{\frac{8\ KT}{\pi\ m^*}}$$

Thermal velocities of electrons and holes are 4.3×10^7 and 1.5×10^7 cm/s, respectively.

Problem 4

Calculate dielectric relaxation time in p-type Ge at room temperature. Assume that all acceptors are ionized. $N_a = 10^{15}$ cm^{-3}, $\varepsilon = 16$, $\mu_p = 1900$ cm^2 V^{-1}s^{-1}.

Solution: (1.6)
$\tau_r = \varepsilon/4\ \pi e\ N_a\ \mu p = 4.7 \times 10^{-12}$ s.

Problem 5
Calculate dielectric relaxation time in intrinsic Si at 300 K. $\varepsilon = 12$, $\mu_n = 1400$ cm^2 V^{-1}s^{-1}, $\mu_n = 3.1\ \mu_p$.

Solution:
In this case,

$$\tau_t = \frac{\epsilon}{4\,\pi\, e\, n_i\,(\mu_n + \mu_p)} = 3.4\ x\ 10^{-7}\ s$$

Problem 6
Calculate the ambipolar diffusion coefficient of intrinsic (undoped) Ge at 300 K. $\mu_n/\mu_p = 2.1$, $\mu_n = 3900$ cm^2 V^{-1}s^{-1}.

Solution:
D = 65 cm^2/s.

Problem 7
Holes are injected into n-type Ge so that at the sample surface $\Delta p_0 = 10^{14}$ cm^{-3}. Calculate Δp at the distance of 4 mm from the surface if $\tau_p = 10^{-3}$ s and $D_p = 49$ cm^2/s.

Solution:

$$\Delta\, p = \Delta p_o\ x\ exp\left(-\frac{L}{\sqrt{D_p \tau_p}}\right) = 1.6\ x\ 10^{13}\ cm^{-3}$$

Problem 8
What is the width of an infinite quantum well if the second lowest energy of a free electron confined to the well equals 100 meV?

Solution

The second lowest energy is calculated from

$$E_2 = \frac{h^2}{2\, m^*}\left(\frac{2}{2\, I_x}\right)^2 = 1.6\, x10^{-20}\, J$$

One can therefore solve for the width, L$_x$, of the well, yielding:

$$I_x = \frac{h}{\sqrt{2\, m^* E_2}} = \frac{6.626\, x\, 10^{-34}}{\sqrt{2\, x 9.11\, x\, 1o^{-31} x 1.6\, x 10^{-20}}} = 3.88\, nm$$

Problem 9

Calculate the lowest three possible energies of an electron in a hydrogen atom in units of electron volt. Identify all possible electron energies between the lowest energy and -2 eV.

Solution

The three lowest electron energies in a hydrogen atom can be calculated from

$$E_n = -\frac{13.6\, eV}{n^2}\ with\, n = 1\, .2\, , and\; 3$$

resulting in:

E_1 = –13.6 eV, E_2 = -3.4 eV and E_3 = -1.51 eV

The second lowest energy, E_2, is the only one between the lowest energy, E_1, and –2 eV.

Problem 9

Calculate the maximum fraction of the volume in a simple cubic crystal occupied by the atoms. Assume that the atoms are closely packed and that they can be treated as hard spheres. This fraction is also called the packing density.

Solution:

The atoms in a simple cubic crystal are located at the corners of the units cell, a cube with side a. Adjacent atoms touch each other so that the radius of each atom equals a/2.

There are eight atoms occupying the corners of the cube, but only one eighth of each is within the unit cell so that the number of atoms equals one per unit cell. The packing density is then obtained from:

$$\frac{\text{volume of atoms}}{\text{volume of unit cell}} = \frac{\frac{4}{3}\ \mathbf{p}\ \mathbf{r}^3}{\mathbf{a}^3} = \frac{\frac{4}{3}\ \mathbf{p}\ \left(\frac{\mathbf{a}}{2}\right)^3}{\mathbf{a}^3} = \frac{\boldsymbol{\rho}}{\mathbf{6}} = \mathbf{52\%}$$

or about half the volume of the unit cell is occupied by the atoms. The packing density of four cubic crystals is listed in the table below.

	Radius	Atoms/ unit cell	Packing density
Simple cubic	$^{a}/_{2}$	1	$^{\rho}/_{6} = 52\%$
Body centered cubic	$\sqrt{3}\,^{a}/_{4}$	2	$^{\rho\sqrt{3}}/_{8} = 68\%$
Face centered cubic	$\sqrt{2}\,^{a}/_{4}$	4	$^{\rho\sqrt{2}}/_{6} = 74\%$
Diamond	$\sqrt{3}\,^{a}/_{8}$	8	$^{\rho\sqrt{3}}/_{16} = 34\%$

Problem 10

Calculate the packing density of the body centered cubic, the face centered cubic and the diamond lattice, listed in example 2.1 p 28.

Solution

The packing density is calculated as obtained from:

$$\frac{\text{volume of atoms}}{\text{volume of unit cell}} = \frac{\frac{4}{3}\ \mathbf{p}\ \mathbf{r}^3}{\mathbf{a}^3}$$

The correct radius and number of atoms per unit cell should be used.

A body centered cubic lattice contains an additional atom in the middle and therefore contains two atoms per unit cell. The atoms touch along the body diagonal, which equals $\sqrt{3}$

a. The radius is one quarter of the body diagonal.
A face centered cubic lattice contains six additional atoms in the center of all six faces of the cube. Since only half of the atoms is within the cube the total number of atoms per unit cell equals four. The atoms touch along the diagonal of the faces of the cube, which equals $\sqrt{2}$a . The radius is one quarter of the diagonal.
The diamond lattice contains two face centered cubic lattice so that the total number of atoms per unit cell equals twice that of the face centered lattice, namely eight. The atoms touch along the body diagonal, where two atoms are one quarter of the body diagonal apart or $\sqrt{3}$ a/4 . The radius equals half the distance between the two atoms.
The radius, number of atoms per unit cell and the packing density are summarized in the table below.

	Radius	Atoms/ unit cell	Packing density
Simple cubic	$a/2$	1	$\rho/6 = 52\%$
Body centered cubic	$\sqrt{3}\,a/4$	2	$\rho\sqrt{3}/8 = 68\%$
Face centered cubic	$\sqrt{2}\,a/4$	4	$\rho\sqrt{2}/6 = 74\%$
Diamond	$\sqrt{3}\,a/8$	8	$\rho\sqrt{3}/16 = 34\%$

Problem 11

Electrons in undoped gallium arsenide have a mobility of 8,800 cm^2/V-s. Calculate the average time between collisions. Calculate the distance traveled between two collisions (also called the mean free path). Use an average velocity of 10^7 cm/s.

Solution The collision time, tc, is obtained from:

$$t_c = \frac{m_n m_e^*}{q} = \frac{0.88\ x\ 0.067\ x\ 9.1\ x\ 10^{-32}}{1.6\ x\ 10^{-19}} = 0.34\ ps$$

$$l = v_{avarage} t_c = 10^7\ x\ o.34\ x\ 10^{-12} = 34\ nm$$

Problem 12

Calculate dielectric relaxation time in p-type Ge at room temperature. Assume that all acceptors are ionized. $N_a = 10^{15}$ cm^{-3}, ϵ = 16, $\mu_p = 1900$ cm^2 V^{-1}s^{-1}.

Solution:

$$\tau_r = \frac{\epsilon}{4\,\pi\,e\,N_a \mu_p} = 4.7\ x\ 10^{-12}\ s$$

Problem 13

Calculate dielectric relaxation time in intrinsic Si at 300 K. ϵ = 12, μ_n = 1400 cm^2 V^{-1}s^{-1}, $\mu_n = 3.1\mu_p$.

Solution:

In this case

$$\tau_r = \frac{\epsilon}{4\,\pi\,e\,n_i\,(\mu_n + \mu_p)} = 3.4\ x\ 10^{-7}\ s$$

Part (2) Solved PROBLEMS conductivity

Example (1)

Calculate the conductivity and the resistivity of n-type silicon wafer which contains 10^{16} electrons per cubic centimeter with an electron mobility of 1400 cm^2/Vs.

Solution:

The conductivity is obtained by adding the product of the electronic charge, q, the carrier mobility, and the density of carriers of each carrier type, or:

$$\sigma = q\,(n\,\mu_n + p\,\mu_p)$$

As n-type material contains almost no holes, the conductivity equals:

$$\sigma = q\, n\, \mu_n = 1.6\ x\ 10^{-19}\ x\ 1400\ x\ 10^{6} = 2.24\ \ 1\Omega cm$$

The resistivity equals the inverse of the conductivity or:

$$\rho = \frac{1}{\sigma} = \frac{1}{q\left(n\,\mu_n + p\,\mu_p\right)}$$

and equals $\rho = 1/\sigma = 1/2.24 = 0.446\ \Omega$ cm.

Example (2)

An n-type piece of silicon of length L = 10 micron has a cross sectional area A= 0.001 cm^2. A voltage V = 10 Volts applied across the sample yielding a current I = 100 mA. What is the resistance, R of the silicon sample, its conductivity, σ , and electron density, n?

μ_n= 1400 cm^2/Vs

Solution The resistance of the sample equals

R = V/I = 10/0.1 = 100 Ω.

Since R = L /(σA)the conductivity is obtained from:

σ = L/(R A) = 0.001/(100 x 0.001) = 0.01 1/ Ω cm.

The required electron density is related to the conductivity by:

$\sigma = q\, n\, \mu_n$ so that the density equals:

$n = \sigma/(q\mu_n) = 0.01/(1.6 \times 10^{-19} \times 1400) = 4.46 \times 10^{13}$cm^{-3}.

Example (3)

A silicon wafer contains 10^{18}cm^{-3}phosphor atoms. Using the data in the table;

	Arsenic	Phosphorus	Boron
$\mu_{min}(m^2V^{-1}s^{-1})$	52.2	68.5	44.9
$\mu_{max}(m^2V^{-1}s^{-1})$	1417	1414	470.5
N_i (cm^{-3})	9.68 x 10^{16}	9.20 x 10^{16}	2.23 x 10^{17}
α	0.68	0.711	0.719

Calculate the resistivity and conductivity of the material. Repeat for arsenic and boron atoms.

Solution

Plugging the values from table into the following equation,

$$\mu = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N}{N_i}\right)\,\alpha}$$

one obtains a mobility of 277 cm^2/V-sec for phosphorus-doped material, 284 cm^2/V-sec for arsenic-doped material and 153 cm^2/V-sec for boron-doped material, corresponding to a resistivity of 22.6, 22.0

and 40.9 mΩ .cm and a conductivity of 44.3, 45.4 and 24.5 1/Ω cm.

Example 4

Estimate the electrical conductivity of intrinsic silicon at 300 K, given that the electron and hole motilities are0.15 m^2 /V-s and 0.05 m^2 /V-s.

Solution

The conductivity arises due to both electrons and holes

$$\sigma = q\, n_i(\mu_e + \mu_h)$$

The intrinsic carrier concentration was calculated to be at 300 K. Thus

$$\sigma = 1.6\ x\ 10^{-19}\ x\ 1.2\ x\ 10^{16}\ x\ 0.2 = 3.84\ x\ \ 10^{-4}\Omega/m$$

Exercise 5

A sample of an intrinsic semiconductor has a band gap of 0.7 eV, assumed independent of temperature.

Taking μ_h = 0.5 μ_e and μ_h = 2 μ_e, find the relationship between the conductivity at 200 K and300 K.

(Ans. ratio of conductivity = 2014.6 eV)

E_F (300 K) - E_F (200 K) = 4.33 x 10^{-3} eV

Example 2.9

A piece of silicon doped with arsenic (N_d = 10^{17} cm^{-3}) is 100 µm long, 10 µm wide and 1 µm thick. Calculate the resistance of this sample when contacted one each end.

Solution

The resistivity of the silicon equals:

$$\rho = \frac{1}{q\, n\, m_x} = \frac{1}{1.6\ x\ 10^{-19}\ x\ 10^{17}\ x\ 727} = 0.086\ \Omega\ cm$$

where the mobility was obtained from Table . The resistance then equals

$$R = \rho \frac{L}{w_t} = 0.086\ x\ \frac{100\ x\ 10^{-4}}{100\ x\ 10^{-4}\ x10^{-4}} = 8.6\ K\Omega$$

$$R_s = \frac{\rho}{t} = \frac{0.086}{10^{-4}} = 860\ \Omega/sq$$

From which one then obtains the resistance:

$$R = R_s \frac{L}{w} = 860\ x\ \frac{100\ x\ 10^{-4}}{10\ x\ 10^{-4}} = 8.6\ K\Omega$$

Problem 2.13

The resistivity of a silicon wafer at room temperature is 5 Ωcm. What is the doping density? Find all possible

solutions.

Solution Starting with a initial guess that the conductivity is due to electrons with a mobility of 1400 cm^2/V-s, the corresponding doping density equals:

$$N_D = n = \frac{1}{q\ \mu_n \rho} = \frac{1}{1.6\ x\ 10^{-19}\ x\ 1400\ x\ 5} = 8.9\ x\ 10^{14} cm^{-3}$$

The mobility corresponding to this doping density equals

$$\mu_n = \mu_{max} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N_D}{N_\rho}\right)^a} = 1366\ cm^2/vs$$

Since the calculated mobility is not the same as the initial guess, this process must be repeated until the assumed mobility is the same as the mobility corresponding to the calculated doping density, yielding:

N_d = 9.12 x 10^{14} cm^{-3} and μ_n = 1365 cm^2/V-s

For p-type material one finds:

N_a = 2.56 x 10^{15} cm^{-3} and μp = 453 cm^2/V-s

Example 2.11

Calculate the electron and hole densities in an n-type silicon wafer (N_d = 10^{17} cm^{-3}) illuminated uniformly with 10 mW/cm^2 of red light (E_{ph} = 1.8 eV). The absorption coefficient of red light in silicon is 10^{-3} cm^{-1}. The minority carrier lifetime is 10 ms.

Solution

The generation rate of electrons and holes equals:

$$G_n = G_p = \propto \frac{P_{OPT}}{E_{ph}\, q} = 10^{-3}\, \frac{10^{-2}}{1.8\, x\, 1.6\, x\, 10^{-19}} = 3.5\, x\, 10^{23}\, cm^{-3} s^{-1}$$

Where, α is absorption coefficient, P_{opt} illumination power , E_{pt} is the red light , where the photon energy was converted into Joules. The excess carrier densities are then obtained from:

$$\delta_n = \delta_p = \tau_p\, G_p = 10\, x\, 10^{-3}\, x\, 3.5\, x\, 10^{23} = 3.5\, x\, 10^{21}\, cm^{-3}$$

The excess carrier densities are then obtained from: So that the electron and hole densities equal:

$$n = n_o + \delta_n = 10^{17} + 3.5\, x\, 10^{21} = 3.5\, x\, 10^{21} cm^{-3}$$

Problem 2.14

How many phosphorus atoms must be added to decrease the resistivity of n-type silicon at room temperature from 1 Ω−cm to 0.1 Ω−cm. Make sure you include the doping dependence of the mobility. State your assumptions.

Solution

Starting with a initial guess that the conductivity is due to electrons with a mobility of 1400 cm²/V-s, the corresponding doping density corresponding to the initial resistivity of 1 Ω−cm equals:

$$N_D \cong n = \frac{1}{q\, \mu_n \rho} = \frac{1}{1.6\, x\, 10^{-19}\, x\, 1400\, x\, 1} = 4.46\, x\, 10^{15} cm^{-3}$$

The mobility corresponding to this doping density equals

$$\mu_n = \mu_{max} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N_D}{N_\rho}\right)^a} = 1274\, cm^2/vs$$

Since the calculated mobility is not the same as the initial guess, this process must be repeated until the assumed

mobility is the same as the mobility corresponding to the calculated doping density, yielding:

$$N_{d,initial} = 4.94 \text{ x } 10^{15} \text{ cm}^{-3} \text{ and } \mu_n = 1265 \text{ cm}^2/\text{V-s}$$

Repeating this procedure for a resistivity of 0.1 Ω−cm one find the final doping density to be

$$N_{d,final} = 8.08 \text{ x } 10^{16} \text{ cm}^{-3} \text{ and } \mu n = 772 \text{ cm}^2/\text{V-s}$$

The added density of phosphorous atoms therefore equals

$$N_{d,\ added} = 4.94 \text{ x } 10^{15} - \ = 7.59 \text{ x } 10^{16} \text{ cm}^{-3}$$

Problem 2.26

A piece of silicon has a resistivity which is specified by the manufacturer to be between 2 and 5 Ohm cm. Assuming that the mobility of electrons is 1400 cm^2/V-sec and that of holes is 450 cm^2/V-sec, what is the minimum possible carrier density and what is the corresponding carrier type? Repeat for the maximum possible carrier density.

Solution

The minimum carrier density is obtained for the highest resistivity and the material with the highest carrier mobility, i.e. the n-type silicon. The minimum carrier density therefore equals:

$$n = \frac{1}{q\,\mu_n \rho_{max}} = \frac{1}{1.6 \ x\ 10^{-19}\ x\ 1400 x\ 5} = 8.92\ x\ 10^{14} cm^{-3}$$

The maximum carrier density is obtained for the lowest resistivity and the material with the lowest carrier mobility, i.e. the p-type silicon. The maximum carrier density therefore equals:

$$p = \frac{1}{q\,\mu_p\,\rho_{max}} = \frac{1}{1.6 \ x\ 10^{-19}\ x\ 450 x\ 2} = 6.94\ x\ 10^{15} cm^{-3}$$

Problem 2.27

A silicon wafer has a 2-inch diameter and contains 10^{14} cm^{-3} electrons with a mobility of 1400 cm^2/V-sec. How thick

should the wafer be so that the resistance between the front and back surface equals 0.1 Ohm?

Solution

The resistance is given by

$$R = \rho\,\frac{L}{A}$$

Where A is the area of the wafer and L the thickness, so that the wafer thickness equals:

$$L = \frac{R\,A}{\rho} = \frac{0.1\;x\;p\;x\;(2.54)^2}{44.6} = \;0.455\,mm$$

The resistivity, ρ, was obtained from:

$$\rho = \frac{1}{q\,n\,\mu_n} = \frac{1}{1.6\;x\;10^{-19}\;x\;1400x\;10^{14}} = 44.6\;\Omega\;cm$$

Part (3) Solved PROBLEMS

Temperature Dependence of Semiconductor Conductivity

Band gap of Si depends on the temperature as $E_g = 1.17\text{ eV} - 4.73 \times 10^{-4}\,T^2 / T + 636$. Find a concentration of electrons in the conduction band of intrinsic (undoped) Si at $T = 77$ K if at 300 K $n_i = 1.05 \times 10^{10}\text{ cm}^{-3}$.

Solution: (1.2)

$$n_i^2 \;=\; N_c N_v\,exp\left(-\frac{E_g}{KT}\right) \approx\; T^3\exp\left(-\frac{E_g}{KT}\right)$$

Therefore,

$$n_1(T_2) =\; n_1(T_1)\left(\frac{T_2}{T_1}\right)^{3/2}\exp\left(-\,\frac{E_g T_2}{2\,K\,T_2} + \frac{E_g T_1}{2\,K\,T_1}\right)$$

Putting the proper values in the formula we obtain that $n_i(77K) \approx 10^{-20}\text{cm}^{-3}$.

5. Hole mobility in Ge at room temperature is 1900 $\text{cm}^2\,\text{V}^{-1}\text{s}^{-1}$.

Find the diffusion coefficient.

Solution: (1.5)
From eD = μkT, it follows that D = 49 cm^2/s.

Example 2.4b
Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at 300, 400, 500 and 600 K.

Solution
The intrinsic carrier density in silicon at 300 K equals:

$$n_i(300K) = \sqrt{N_c N_v}\, exp\left(\frac{-E_g}{2\,KT}\right)$$
$$= \sqrt{2.81\ x\ 10^{19}\ x\ 1.83\ x\ 10^{19}}\, exp\ \left(\frac{-1.12}{2\ x\ 0.0258}\right)$$
$$= 8.72\ x\ 10^9 m^{-3}$$

Similarly, one finds the intrinsic carrier density for germanium and gallium arsenide at different temperatures, yielding:

	Germanium	Silicon	Gallium Arsenide
300 K	2.02 x 10^{13}	8.72 x 10^9	2.03 x 10^6
400 K	1.38 x 10^{15}	4.52 x 10^{12}	5.98 x 10^9
500 K	1.91 x 10^{16}	2.16 x 10^{14}	7.98 x 10^{11}
600 K	1.18 x 10^{17}	3.07 x 10^{15}	2.22 x 10^{13}

Example 2.2 Calculate the energy band gap of germanium, silicon and gallium arsenide at 300, 400, 500 and 600 K.

Solution The band gap of silicon at 300 K equals:

$$E_g(300\ k) = E_g(0\ k) - \frac{a\,T^2}{T+b} = 1.166 - \frac{0.473\ x\ 10^{-3}\ x\ (300)^2}{300+636}$$
$$= 1.12\ eV$$

Similarly, one finds the energy band gap for germanium and

gallium arsenide, as well as at different temperatures, yielding:

	Germanium	**Silicon**	**Gallium Arsenide**
T = 300 K	**0.66 eV**	**1.12 eV**	**1.42 eV**
T = 400 K	**0.62 eV**	**1.09 eV**	**1.38 eV**
T = 500 K	**0.58 eV**	**1.06 eV**	**1.33 eV**
T = 600 K	**0.54 eV**	**1.03 eV**	**1.28 eV**

Problem 2.2 At what temperature does the energy band gap of silicon equal exactly 1 eV?

Solution

The energy band gap is obtained from:

$$E_g(T) = E_g(0\,k) - \frac{a\,T^2}{T+b} = 1.166 - \frac{0.473\ x\ 10^{-3}\ x\ T^2}{T+636} = 1\ eV$$

The type equation here **is quadratic equation can be solved yielding**

$$T = \frac{E_g(0\,k) - E_g(T\,k)}{2\,a} + \sqrt{\left(\frac{E_g(0\,k) - E_g(T\,k)}{2\,a}\right)^2 + \frac{b\,\left(E_g(0\,k) - E_g(T\,k)\right)}{a}} = 679\ K$$

Problem (6)

If no electron-hole pairs were produced in germanium (Ge) until the temperature reached the value corresponding to the energy gap, at what temperature would Ge become conductive? (E_{th} = 3/2 kT)

Solution

$$E_{th} = \frac{3\,KT}{2}\ , E_g = 0.72 x\ 1.6\ x\ 10^{-19}\ J$$

$$T = \frac{0.72\ x\ 1.6\ x\ 10^{-19}\ x\ 2}{3\ x\ 1.38\ x\ 10^{-23}} = 5565\ K = 5.3\ x\ 10^3\ {}^oC$$

The temperature would have to be 5.3 x 10^{3}°C, about 4400°C above the melting point of Ge.

Example (3)

Hole mobility in Ge at room temperature is 1900 cm^{2} V^{-1}s^{-1}. Find the diffusion coefficient.

Solution:
From eD = μkT, it follows that D = 49 cm^{2}/s.

8. How does the reverse current of a Si *p-n* junction change if the temperature raises from 20 to 50 °C? The same for a Ge *p-n* junction. Band gaps of Si and Ge are 1.12 and 0.66 eV, respectively.

Solution: (3.8)
Since

$$J_s \sim n_i^2 \sim T^3\, exp\left(-\frac{E_g}{KT}\right),$$

we get

$$\frac{J_s(T_2)}{J_s(T_1)} = \left(\frac{T_2}{T_1}\right)^3 exp\left(-\frac{E_g}{K\,T_2} + \frac{E_g}{K\,T_1}\right)$$

From here the ratios of the reverse currents in the *p-n* junctions made of Ge and Si are 15 and 82, respectively.

9. Estimate temperatures at which *p-n* junctions made of Ge, Si, and GaN lose their rectifying characteristics. In all cases $N_a = N_d = 10^{15}$ cm^{-3}. Assume that E_g are independent of the temperature and are 0.66, 1.12, and 3.44 eV for Ge, Si, and GaN, respectively. Intrinsic carrier concentrations at room temperature are $n^{Ge}{}_i = 2 \times 10^{13}$, $n^{Si}{}_i = 10^{10}$, and $n^{GaN}{}_i = 10^{-9}$ cm^{-3}.

Solution: (3.9)
***p-n* junction stops working when concentrations of electrons**

and holes equalize. It happens when $N_d(N_a) \approx n_i = \sqrt{N_c N_v}$ $\exp(-E_g/2kT) \approx T^{3/2} \exp(-E_g/2kT)$.

From here and the parameters given we get that the maximum temperatures are $T_{Ge} \approx 400$ K, $T_{Si} \approx 650$ K, and $T_{GaN} \approx 1700$ K. That is, only wide band gap semiconductors are suitable for extremely applications.

Part (4) Solved PROBLEMS

Electron and Hole Concentrations

Example 2.6a

A germanium wafer is doped with a shallow donor density of 3 ni/2. Calculate the electron and hole density.

Solution

The electron density is obtained from equation

$$n_o = \frac{N_D^+ - N_A^-}{2} + \sqrt{\left(\frac{N_D^+ - N_A^-}{2}\right)^2 + n_i^2} = n_i\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right)$$

$$= 2\, n_i$$

and the hole density is obtained using the mass action law:

$$p_o = \frac{n_i^2}{n_o} = \frac{n_i}{2}$$

Example 2.6b

A silicon wafer is doped with a shallow acceptor doping of 10^{16} cm^{-3}. Calculate the electron and hole density.

Solution

Since the acceptor doping is much larger than the intrinsic density and much smaller than the effective density of states, the hole density equals: -

$$p_o \cong N_a^+ = 10^{16} cm^{-2}$$

The electron density is then obtained using the mass action law

$$n_o = \frac{n_i^2}{N_D^+} = \frac{10^{20}}{10^{16}} = 10^4 cm^{-3}$$

Problem 2.6

Calculate the effective density of states for electrons and holes in germanium, silicon and gallium arsenide at room temperature and at 100 °C. Use the effective masses for density of states calculations.

Solution The effective density of states in the conduction band for germanium equals:

$$N_c = 2\left(\frac{2\,p\,m_e^*\,KT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\,p\,x\,0.55\,x\,9.11\,x\,10^{-31}\,x\,1.38\,x\,10^{-22}\,x\,300}{(6.626\,x\,10^{-34})^2}\right)^{3/2}$$

$$= 1.02\,x\,10^{25} m^{-3} = 1.02\,x\,10^{19} cm^{-3}$$

Where, the effective mass for density of states is used. Similarly, one finds the effective densities for silicon and gallium arsenide and those of the valence band, using the effective masses listed below:

	Germanium	Silicon	Gallium Arsenide
m_e/m_0	0.55	1.08	0.067
Nc (cm^{-3})	1.02×10^{19}	2.81×10^{19}	4.35×10^{17}
Nv (cm^{-3})	5.64×10^{18}	1.83×10^{19}	7.57×10^{18}

The effective density of states at 100 °C (372.15 K) are obtain from:

$$N_c(T) = N_c(300k)\left(\frac{T}{300}\right)^{3/2}$$

yielding:

T = 100°C	Germanium	Silicon	Gallium Arsenide
Nc (cm^{-3})	1.42×10^{19}	3.91×10^{19}	6.04×10^{17}
Nv (cm^{-3})	7.83×10^{18}	2.54×10^{19}	1.05×10^{18}

Problem 2.7

Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at room temperature (300 K). Repeat at 100 °C. Assume that the energy band gap is independent of temperature and use the room temperature values.

Solution The intrinsic carrier density is obtained from:

$$n_i(T) = \sqrt{N_c V_v}\, exp\left(-\frac{E_g}{2KT}\right)$$

where both effective densities of states are also temperature dependent. Using the solution of Problem 2.6 one obtains

T = 300 K	Germanium	Silicon	Gallium Arsenide
ni (cm^{-3})	2.16 x 10^{13}	8.81 x 10^{9}	1.97 x 10^{6}
T = 100°C	Germanium	Silicon	Gallium Arsenide
ni(cm^{-3})	3.67 x 10^{14}	8.55 x 10^{11}	6.04 x 10^{8}

Example 2.11

Calculate the electron and hole densities in an n-type silicon wafer (N_d = 10^{17} cm^{-3}) illuminated uniformly with 10 mW/cm^2 of red light (E_{ph} = 1.8 eV). The absorption coefficient of red light in silicon is 10^{-3} cm^{-1}. The minority carrier lifetime is 10 µs.

Solution

The generation rate of electrons and holes equals:

$$G_n = G_p = a\,\frac{p_{opt}}{E_{ph}\,A} = 10^{-3}\,\frac{10^{-2}}{1.8\,x\,1.6\,x\,10^{-19}}$$
$$= 3.5\,x\,10^{13} cm^{-3} s^{-1}$$

where the photon energy was converted into Joules. The excess carrier densities are then obtained from

$$dn = dp = t_p G_p = 10\,x\,10^{-6}\,x\,3.5\,x\,10^{13} = 3.5\,x\,10^{8} cm^{-3}$$

So that the electron and hole densities equal:

$$n = n_o + dn = 10^{17} + 3.5\,x\,10^{8} = 10^{17} cm^{-3}$$

$$p = \frac{n_i^2}{n_o} + dp = \frac{(10^{10})^2}{10^{17}} + 3.5\,x\,10^{8} = 3.5\,x\,10^{8} cm^{-3}$$

Problem 2.11

A silicon wafer contains 10^{16} cm^{-3} electrons. Calculate the hole density and the position of the intrinsic energy and the Fermi energy at 300 K. Draw the corresponding band diagram to scale, indicating the conduction and valence band edge, the intrinsic energy level and the Fermi energy level. Use $n_i = 10^{10}$ cm^{-3}.

Solution The hole density is obtained using the mass action law:

$$p = \frac{n_i^2}{n} = \frac{10^{20}}{10^{16}} = 10^{4}\,cm^{-3}$$

The position of the intrinsic energy relative to the midgap energy equals:

$$E_i - \frac{E_c + E_v}{2} = -\frac{3}{4} KT\ ln\ \frac{m_h^*}{m_e^*} = \frac{3}{4}\,x\,0.0258\ ln\ \frac{0.81}{1.08}$$
$$= 5.58\ meV$$

The position of the Fermi energy relative to the intrinsic energy equals:

$$E_F - E_i = KT\ ln\left(\frac{N_d}{n_i}\right) = 0.0258\ ln\ \frac{10^{16}}{10^{10}} = 357\ meV$$

Problem 2.12

A silicon wafer is doped with 10^{13} cm^{-3} shallow donors and 9 x 10^{12} cm^{-3} shallow acceptors. Calculate the electron and hole density at 300 K. Use $n_i = 10^{10}$ cm^{-3}.

Solution

Since there are more donors than acceptors, the resulting material is n-type and the electron density equals the difference between the donor and acceptor density or:

$$n = N_d - N_a = 10^{13} - 9\,x\,10^{12} = 10^{12} cm^{-3}$$

The hole density is obtained by applying the mass action law:

$$p = \frac{n_i^2}{n} = \frac{10^{20}}{10^{12}} = 10^8\, cm^{-3}$$

Example (1)

A Si sample at room temperature is doped with 10^{11}As atoms/cm^3. What are the equilibrium electron and hole concentrations at 300 K?

Solution Since the $N_A$is zero we can write,

$$n_o p_o = n_i^2$$

$$and\;\; n_o + N_A = p_o + N_D$$

$$n_o^2 - N_D - n_i^2 = o$$

Solving this quadratic equation results in

n_0= 1.02x10^{11}[cm^{-3}]

and thus,

p_0= n_i^2/ n_0= 2.25x10^{20}/ 1.02x10^{11}

p_0= 2.2x10^9[cm^{-3}]

Notice that, since N_D>n_i, the results would be very similar if we assumed n_o=N_D=10^{11}cm^{-3}, although there would be a slight error since $N_D$is not much greater than n_i.

<u>Question 2</u>:

What are the carrier concentrations and Conductivity in intrinsic Si?[For Si: ε_g = 1.1 eV, m_e^* = 0.25m_e, m_h^* = 0.5m_e, μ_e = 0.15 m^{2}V^{-1}s^{-1}, μ_h = 0.05 m^{2}V^{-1}s^{-1}]

Solution:

For intrinsic material ,

$n_c = p_v = n_i$, Obtain n_i from:

$$\mathbf{n_i} = 2\left(m_e^* m_h^*\right)^{\frac{3}{4}}\left(\frac{kT}{\pi\hbar^2}\right)^{\frac{3}{2}}\exp\left\{-\frac{\varepsilon_g}{2kT}\right\}$$

For Si: $\varepsilon_g = 1.1$ **eV,** $m_e^* = 0.25m_e$, $m_h^* = 0.5m_e$

So at room temperature (T = 300K):

$$n_i = 2\,(0.25\,m_e\,x\,0.5\,m_e)^{3/4}\,x\,\left\{\frac{k\,x\,300}{3.14\,xh^2}\right\}^{3/2}\,x$$

$$\exp\left\{-\frac{1.1\,eV}{2\,x\,300\,k}\right\} = 9.8\,x\,10^{15}m^{-3}$$

Conductivity $\sigma = n_c e\mu_e + p_v e\mu_h$

$$\sigma = n_i e\mu_e + n_i e\mu_h$$

$$\sigma = \left(9.8\,x\,10^{15}m^{-3}\,x\,1.6\,x10^{-19}\,x\,0.15\,m^2/\mathrm{Vs}\right)$$

$$+\left(9.8\,x\,10^{15}m^{-3}\,x\,1.6\,x10^{-19}\,x\;0.05\,m^2/\mathrm{Vs}\right)$$

$$\sigma = (2.3 \times 10^{-4}) + (7.8 \times 10^{-5})\,\Omega^{-1}m^{-1}$$

$$= (2.3 + 0.78)\,x\,10^{-4}$$

$$\sigma = 3.1 \times 10^{-4}\Omega^{-1}m^{-1}$$

Comparing answers for Q1 and Q2:

doping Si with 1 part in 10^6 (of P) has led to an increase in σ of a factor > than 10^6.

Example 10 (3)

Pure germanium has a band gap of 0.67 eV. It is doped with 3 x 10^{21}/ m^3 of donor atoms. Find the densities of electrons and holes at 300 K. (effective masses $m_e = 0.55\ m_o$ and $m_h = 0.37\ m_o$).

Solution:

For Ge , the intrinsic concentration is

$$n_i = \sqrt{N_c N_v} e^{-E_g/_{2kT}}$$

From tables, we find for pure germanium

N_C = 1 x 10^{25} and N_V = 6 x 10^{24} , E_g = 067 eV

Substituting

$$n_i = \sqrt{1x\ 10^{25}\ x\ 6x\ 10^{24}} e^{-0.67/_{2\ x\ 0.026}}$$

Given numerical values, n_i = 2.4 x 10^{19} / m^3

The density of donor atoms is N_D = 3 x 10^{21}/m^3.

Thus the electron density is given by:

$$n = \frac{N_d}{2} + \sqrt{\frac{N_d^2 + 4\, n_i^2}{4}} \cong N_d \quad as\ N_d >> n_i$$

Substituting,

$$n = \frac{3x10^{21}}{2} + \sqrt{\frac{(3x10^{21})^2 + 4\,(2.4x10^{19})^2}{4}}$$

Thus, n = 3 x 10^{21}/m^3 , using n_i^2 = n p , we get for the density of holes $(2.4 \times 10^{19})^2$ = 3 x 10^{21}p , then p =$(2.4 \times 10^{19})^2$/3 x 10^{21}=

p = 1.92 x 10^{17}/m^3.

Problem (4)

The resistivity of a silicon wafer at room temperature is 5 Ω cm. What is the doping density? Find all possible solutions.

Solution:

Starting with an initial guess that the conductivity is due to electrons with a mobility of 1400 cm2/V-s, the corresponding doping density equals:

$$N_D \cong n = \frac{1}{q\,\mu_n\,\rho} = \frac{1}{1.6\,x\,10^{-19}\,x\,1400\,x\,5} = 8.9\,x\,10^{14}cm^{-3}$$

The mobility corresponding to this doping density equals

$$\mu_n = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N_d}{N_r}\right)^{\propto}}$$

Since the calculated mobility is not the same as the initial guess, this process must be repeated until the assumed mobility is the same as the mobility corresponding to the calculated doping density, yielding: $N_d = 9.12 \times 10^{14}$ cm^{-3} and $\mu_n = 1365$ cm^2/V-s. For p-type material one finds: $N_a = 2.56 \times 10^{15}$ cm^{-3} and $\mu_p = 453$ cm^2/V-s

$N_d = 9.12 \times 10^{14}$ cm^{-3} and $\mu_n = 1365$ cm^2/V-s For p-type material one finds: Na = 2.56×10^{15} cm^{-3} and $\mu_p = 453$ cm^2/V-s

Problem (5)

Consider the problem of finding the doping density, which results in the maximum possible resistivity of silicon at room temperature. ($n_i = 10^{10}$ cm^{-3}, $\mu_n = 1400$ cm^2/V-sec and $\mu_p = 450$ cm^2/V-sec.) Should the silicon be doped at all or do you expect the maximum resistivity when dopants are added? If the silicon should be doped, should it be doped with acceptors or donors (assume that all dopant is shallow). Calculate the maximum resistivity, the corresponding electron and hole density and the doping density.

Solution

Since the mobility of electrons is larger than that of holes, one expects the resistivity to initially decrease as acceptors are added to intrinsic silicon. The maximum resistivity (or minimum conductivity) is obtained from:

$$\frac{d\sigma}{dn} = q\,\frac{d\left(n\,\mu_n + p\mu_p\right)}{dn} = q\,\frac{d\left(n\,\mu_n + {n_i^2\mu_p}/{n}\right)}{dn} = 0$$

Which yields,

$$n = n_i \sqrt{\frac{\mu_p}{\mu_n}} = 0.57\, n_i = 5.7\ x\ 10^9 cm^{-3}$$

The corresponding hole density equals

$p = 1.76\ n_i = 1.76 \times 10^9\ cm^{-3}$

And the amount of acceptors one needs to add equals

$N_a = 1.20\ n_i = 1.20 \times 10^9\ cm^{-3}$.

The maximum resistivity equals:

$$\rho_{max} = \frac{1}{q\,(n\,\mu_n + p\mu_p)} = \frac{1}{q\,n_i(\mu_n + \mu_p)}$$

$$= \frac{1}{1.6\ x\ 10^{-19}\ \ x\ 10^{10}(1400 + 450)} = 394\ k\Omega.cm$$

Problem (6)

The electron density in silicon at room temperature is twice the intrinsic density. Calculate the hole density, the donor density and the Fermi energy relative to the intrinsic energy. Repeat for $n = 5\ n_i$ and $n = 10\ n_i$. Also repeat for $p = 2\ n_i$, $p = 5\ n_i$ and $p = 10\ n_i$, calculating the electron and acceptor density as well as the Fermi energy relative to the intrinsic energy level.

Solution:

The hole density is obtained using the mass action law:

$$p = n_i^2/n$$

The doping density is obtained by requiring charge neutrality

$$N_d - N_a = n - p$$

The Fermi energy is obtained from:

$$E_F - E_i = kT\ \ln(n/n_i)$$

yielding:

	$n = 2\ n_i$	$n = 5\ n_i$	$n = 10\ n_i$
p	$n_i/2$	$n_i\ /5$	$n_i/10$
Nd - Na	$1.5\ n_i$	$4.8\ n_i$	$9.9\ n_i$

EF - Ei	kT ln(2)	kT ln(5))	kT ln(10)
	$p = 2\ n_i$	$p = 5\ n_i$	p = 10 ni
n	$n_i /2$	$n_i /5$	$n_i /10$
Nd - Na	$-1.5\ n_i$	$-4.8\ n_i$	$-9.9\ n_i$
EF - Ei	-kT ln(2)	-kT ln(5)	-kT ln(10)

Exercise 7

A sample of Ge at 300 K is doped with $3 \times 10^{21}/m^3$ of donor atoms and $4 \times 10^{21}/m^3$ acceptor atoms. Find the densities of electrons and holes at 300 K.
(answer, $n = 5.76 \times 10^{17}/m^3$, $p = 10^{21}/m^3$)

Problem (8)

(a) The lattice constant of GaAs is 5.65 A, Determine the number of Ga atoms and As atoms per cm^3. (b) Determine the volume density of germanium atoms in a germanium semiconductor. The lattice constant of germanium is 5.65 A.

Solution:

(a) 4 Ga atoms per unit cell Density=$\frac{4}{(5.65\ x\ 10^{-8})^3}$

Density of Ga=$2.22\times10^{22}cm^{-3}$ 4 As atoms per unit cell, so that Density of As=$2.22\times10^{22}cm^{-3}$

(b) 8 Ge atoms per unit cell Density= $\frac{8}{(5.65\ x\ 10^{-8})^3}$

→ Density of Ge=$4.44\times10^{22}cm^{-3}$

Problem (9)

Calculate the density of valence electrons in silicon.

Solution:

Density or silicon atoms= $5\times10^{22}cm^{-3}$ and 4 valence electrons per atom,
So Density of Valence electrons = $4 \times 5\times10^{22}cm^{-3}= 2 \times 10^{23}cm^{-3}$

Problem 10

(a) Determine the amount (in grams) of boron (B) that, substitution ally incorporated into 1 kg of germanium (Ge), will establish a charge carrier density of 3.091 x 10^{17}/cm^3.

(b) Draw a schematic energy band diagram for this material, and label all critical features.

Solution

(a) The periodic table gives the molar volume of Ge as 13.57 cm^3 and 1 mole of Ge weighs 72.61 g, so set up the ratio

$$\frac{\mathbf{72.61}}{\mathbf{13.57}} = \frac{\mathbf{1000}\ \boldsymbol{g}}{\boldsymbol{x}}$$

and solve for x to get 187.30 cm^3 for the total volume.

The addition of boron gives 1 charge carrier/B atom.

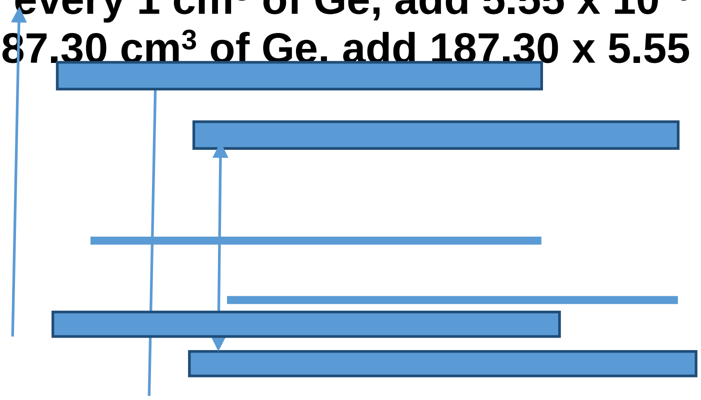
B concentration in Si must be 3.091 x 10^{17} B/cm^3

N_A of B atoms weighs 10.81 g

$$\therefore 3.091\ x\ 10^{17}\ B\ atoms\ weigh = \frac{3.091\ x\ 10^{17}}{6.02\ x\ 10^{23}}\ x\ 10.81 = 5.55\ x\ 10^{6}\ g$$

∴ for every 1 cm^3 of Ge, add 5.55 x 10^{-6} g

for 187.30 cm^3 of Ge, add 187.30 x 5.55 x 10^{-6} = 1.04 x 10^{-3} g B



Conduction band

E$_g$

B$^+$ ………..B$^+$ ………….. B$^+$ 3.091X10^{17}B ions in the band

Acceptor level for B

Valance band3.091X10^{17}holes in the V.B

Problem (11)

The number of electron-hole pairs in intrinsic germanium (Ge) is

given by: $\boldsymbol{n_i = 9.7\,x\,10^{15} T^{3/2} e^{-E_g/_{2\,KT}}{}^{cm^3} \left(E_g = 0.72\,eV\right)}$

(a) What is the density of pairs at T = 20°C?
(b) Will undoped Ge be a good conductor at 200°C? If so, why?

Solution

- **(a) Recall: T in thermally activated processes is the absolute temperature: T °K = (273.16 + t °C); Boltzmann's constant = k = 1.38 x 10^{-23} J/oK**

T = 293.16K:

$$\boldsymbol{n_i = 9.7\,x\,10^{15}\,x\,(293.16)^{3/2} e^{-0.72\,x\,1.6\,x\,10^{-19}/_{2\,x\,1.38\,x\,10^{-23}\,x\,293.16}}{}^{cm^3}}$$
$$\boldsymbol{= 9.7\,x\,10^{15}\,x\,5019\,x\,6.6\,x\,10^{-7} = 3.21\,x\,10^{13}/\,cm^3}$$

- **(b) 200 °C = 473.16 K**

$$\boldsymbol{n_i = 9.7\,x\,10^{15}\,x\,(473.16)^{3/2} e^{-0.72\,x\,1.6\,x\,10^{-19}/_{2\,x\,1.38\,x\,10^{-23}\,x\,473.16}}{}^{cm^3}}$$
$$\boldsymbol{= 9.7\,x\,10^{15}\,x\,1{,}03\,x\;10^{4} x\,1.47\,x\,10^{-4} = 1.47\,x\,10^{16}/\,cm^3}$$

The number of conducting electrons (in the conduction band) at 200°C is by about five orders of magnitude less than that of a good conductor. The material will not be a good conductor. (There are additional factors which contribute to the relatively poor conductivity of Ge at this temperature.)

Problem (12)
Band gap of Si depends on the temperature as

$$\boldsymbol{E_g = 1.17\,eV - 4.73\,x\,10^{-4}\,\frac{T^2}{T + 636}}$$

Find a concentration of electrons in the conduction band of intrinsic (un doped) Si at T = 77 K if at 300 K n$_i$ = 1.05×10^{10} cm^{-3}.

Solution:

$$\boldsymbol{n_i^2 = N_c N_v\,exp\left(-\frac{E_g}{KT}\right) \approx T^3\,exp\left(-\frac{E_g}{KT}\right),therefore}$$

$$n_i(T_2) = n_i\ (T_1)^{3/2} exp\left(-\frac{E_g\,(T_2)}{2KT_2} + \frac{E_g\,(T_1)}{2KT_1}\right)$$

Putting the proper values in the formula we obtain that $n_i(77K) \approx 10^{-20}cm^{-3}$.

Problem (13)

(a) Assume the mobility ratio $\mu_n / \mu_p = b$ in Si is a constant independent of impurity concentration. Find the maximum resistivity p, in terms of the intrinsic resistivity p_i at 300 K. If b = 3 and the hole mobility of intrinsic Si is 450 cm^2/V-s, calculate ρ_i and p_n.

(b) Find the electron and hole concentration, mobility, and resistivity of a GaAs sample at 300 K with $5X10^{15}$ zinc atoms/cm^3, 10^{17} sulfur atoms/cm^3, and 10^{17} carbon atoms/cm^3.

Problem (14)

Consider a compensated n-type silicon at T= 300 K, with a conductivity of σ= 16 S/cm and an acceptor doping concentration of $10^{l7}cm^{-3}$. Determine the donor concentration and the electron mobility. (A compensated semiconductor is one that contains both donor and acceptor impurity atoms in the same region.)

Exercise (2)

For a two band model of silicon, the band gap is 1.11 eV. Taking the effective masses of electrons and holes as m_e = 1.08 m_o and m_h = o.81 m_o , calculate the intrinsic carrier concentration in silicon at 300 K. (KT=0.026 eV)

Solution:

Applying the equation

$$n_i = \frac{1}{4}\left(\frac{2\,KT}{\pi\,h^2}\right)^{3/2} (m_e m_h)^{3/4}\, e^{-\Delta/2\,KT} \qquad (D)$$

Then,

$$n_i = \frac{1}{4}\left(\frac{2\ x\ 0.026}{3.14\ h^2}\right)^{3/2} (1.08\ m_o \ \mathrm{x}\ 0.81\ m_o)^{3/4}\, e^{-1.11/0.026}$$
$$= 1.2\ x\ 10^{16} m^{-3}$$

Example (4) Carrier Concentrations

What is the hole concentration in an N-type semiconductor with 10^{15}cm^{-3} of donors? (n_i is a strong function of E_g and T according to Eq. $n_i = \sqrt{N_c N_v} e^{-E_g/2KT}$, but is independent of the dopant concentration. n_i at room temperature is roughly 10^{10}cm^{-3} for Si and 10^7cm^{-3} for GaAs, which has a larger band gap than Si. For silicon, the np product is therefore 10^{20}cm^{-6} regardless of the conductivity type (P type or N type) and the dopant concentrations).

Solution: For each ionized donor, an electron is created. Therefore, n = 10^{15}cm^{-3}.

$$p = \frac{n_i^2}{n} = \frac{10^{20} cm^{-3}}{10^{15} cm^{-3}} = 10^5 cm^{-3}$$

With a modest temperature increase of 60°C, n remains the same at 10^{15}cm^{-3}, while p increases by about a factor of 2300 because n_i^2 increases according to Eq.

$$n_i = \sqrt{N_c N_v} e^{-E_g/2KT}$$

Example (5)

What is n if p = 10^{17}cm^{-3} in a P-type silicon wafer?

Solution:

$$n = \frac{n_i^2}{p} = \frac{10^{20} cm^{-3}}{10^{17} cm^{-3}} = 10^3 cm^{-3}$$

Part (5) Solved PROBLEMS

The Concept of Mobility , FIELD DEPENDENCE

Problem 2.28 –(1)

Electrons in silicon carbide have a mobility of 1000 cm^2/V-sec. At what value of the electric field do the electrons reach a velocity of 3 x 10^7 cm/s? Assume that the mobility is constant and independent of the electric field. What voltage is required to obtain this field in a 5 micron thick region? How much time do the electrons need to cross the 5 micron thick region?

Solution:

The electric field is obtained from the mobility and the velocity:

$$\boldsymbol{\varepsilon = \frac{\mu}{\nu} = \frac{1400}{3\ x\ 10^7} = 30\ kV/cm}$$

Combined with the length one finds the applied voltage.

$V = \varepsilon L = 30{,}000 \text{ x } 5 \text{ x } 10^{-4} = 15$ V

The transit time equals the length divided by the velocity:

$t_r = L/v = 5 \text{ x } 10^{-4} / 3 \text{ x } 10^7 = 16.7$ ps

1 Thermal Velocity

EXAMPLE 2

What are the approximate thermal velocities of electrons and holes in silicon at room temperature?

SOLUTION:

Assume T = 300 K and recall $m_n = 0.26\ m_0$.

$$Kinetic\ energy = \frac{1}{2}\, m_n\, V_{th}^2 = \frac{3}{2}\, KT$$

$$V_{th} = \sqrt{\frac{3\ KT}{m}} = \left[\left(3\ x\ 1.38\ x\ 10^{-23}\ {}^{J}/_{K}\right)\ x\ \left(\frac{300\ K}{0.26}\ x\ 9.1\ x\ 10^{-31}\ kg\right)\right]^{1/2}$$

$$= 2.3\ x\ m^5\ {}^{m}/_{s} = 2.3\ x\ 10^7\ {}^{cm}/_{s}$$

Note that 1 J = 1 kg·m$_2$/s$_2$. Using m_p = 0.39 m_0 instead of m_n, one would find the hole thermal velocity to be 2.2 × 10^7cm/s. So, the typical thermal velocity of electrons and holes is 2.5 × 10^7cm/s, which is about 1000 times slower than the speed of light and 100 times faster than the sonic speed.

Part (6) Solved PROBLEMS

complete ionization

8. Find Debye length in p-type Ge at 300 K if $N_a = 10^{14}$ cm^{-3}. Assume that all acceptors are ionized, ε = 16.

Solution: (1.8)
L_D = 0.48 μm.

Example 2.5
Calculate the ionization energy for shallow donors and acceptors in germanium and silicon using the hydrogen-like model.
Solution
Using the effective mass for conductivity calculations (Appendix 3) one finds the ionization energy for shallow donors in germanium to be:

$$E_c - E_D = 13.6\,\frac{m^*_{cond}}{m_0 e_r^2}\,eV = 13.6\,\frac{0.12}{(16)^2}\,eV = 6.4\,meV$$

The calculated ionization energies for donors and acceptors in germanium and silicon are provided below.

	Germanium	Silicon
donors	6.4 meV	13.8 meV
acceptors	11.2 meV	20.5 meV

Note that the actual ionization energies differ from this value and depend on the actual donor atom

Example (1)

A Si sample is doped with 10^{-4} atomic% of P donors. Assuming complete ionization of donors at room temperature, calculate the charge carrier concentration and conductivity at room temperature. [For Si: ρ= 2330 kg m^{-3}, atomic weight = 28, μ_e= 0.15 $m^2V^{-1}s^{-1}$, μ_h= 0.05 $m^2V^{-1}s^{-1}$, n_i= $1.5x10^{10}$carriers per cm^3]

Solution:

1) Calculate the fraction of donor atoms (phosphorus atoms per silicon atom) where N_{Si}–number of Si atoms per unit volume

$$\frac{N_D}{N_{Si}} = 10^{-6}$$

Calculate the number of silicon atoms per unit volume

$$N_{Si} = \frac{\rho}{A_{Si}} x\, N_{Avogadro} = \frac{2330\, kg\, m^{-3}}{28\, x\, 10^{-3}\, kg\, mol^{-1}}\, x\, 6\, x\, 10^{23}\, atoms.mol^{-1}$$
$$= 5\, x\, 10^{28}\, Si - atom\, m^3$$

3) Calculate the number of donors atoms (phosphorus)

$$N_D = 5\, x\, 10^{22}\, p - atom\, m^3$$

4) As N_A=0 and N_D>>n_i, then we can safely assume that n_o=N_D and p_o is very small ~ zero

$$\sigma = n_o\, x\, q\, x\, \mu_e$$
$$\sigma = (5x10^{22}p - atom\, m^3)\, x\, (1.6\, x10^{-19}C)\, x\, (0.15\, m^2V^{-1}s^{-1})$$
$$= 1200\, \Omega^{-1}m^{-1}$$

Example (2)

A Si sample is doped with 10^{-4} atomic% of P donors. Assuming complete ionisation of donors at room temperature, calculate the charge carrier concentration and conductivity at room temperature. [For Si: ρ = 2330 kg m^{-3}, atomic weight = 28, μ_e = 0.15 $m^2V^{-1}s^{-1}$, Avogadro's No= 6 x 10^{23}]

Answer 1

$$\frac{N_d}{N_{Si}} = 10^{-6}$$

Where N_{Si} – number of Si atoms per unit volume

Obtain N_{Si} from

N_{Si} = Resistivity x Avogadro's No. / Atomic weight

$$N_{Si} = \frac{\rho}{28 \times 10^{-3}} \times 6 \times 10^{23}$$

$$\boldsymbol{N_{Si} = \frac{2330}{28\ x\ 10^{-3}}\ x\ 6\ x\ 10^{23}}$$

This gives $N_{Si} = 5 \times 10^{28}\ m^{-3}$

So $N_d = N_{Si}\ x\ 10^{-4}\ \% = 5 \times 10^{28}\ m^{-3}\ x\ 10^{-6}$

$N_d = 5 \times 10^{22}\ m^{-3}$.

Complete ionisation, n-type semiconductor:

So charge carrier concentration is

$n_c = N_d = 5 \times 10^{22}\ m^{-3}$

(Neglect p_v) , then Conductivity σ

$\sigma = n_c\, e\, \mu_e$[hole contribution negligible]

So

$\sigma = 5 \times 10^{22} \times 1.6 \times 10^{-19} \times 0.15\ \Omega^{-1}m^{-1} = 1200\ \Omega^{-1}m^{-1}$

Example (6)

Find Debye length in p-type Ge at 300 K if $N_a = 10^{14}\ cm^{-3}$. Assume that all acceptors are ionized, $\epsilon = 16$.

Solution:

$L_D = 0.48\ \mu m$.

9. Calculate the ambipolar diffusion coefficient of intrinsic (un doped) Ge at 300 K. $\mu_n/\mu_p = 2.1$, $\mu_n = 3900\ cm^2\ V^{-1}s^{-1}$.

Solution:
D = 65 cm²/s.

10. Holes are injected into n-type Ge so that at the sample surface $\Delta p_0 = 10^{14}$ cm^{-3}. Calculate Δp at the distance of 4 mm from the surface if $\tau_p = 10^{-3}$ s and $D_p = 49$ cm²/s.

Solution:

$$\Delta p = \Delta p_o \, exp\left(-\frac{L}{\sqrt{D_p \tau_p}}\right) = 1.6 \, x \, 10^{13} cm^{-3}$$

Problem 2.20

The expression for the Bohr radius can also be applied to the hydrogen-like atom consisting of an ionized donor and the electron provided by the donor. Modify the expression for the Bohr radius so that it applies to this hydrogen-like atom. Calculate the resulting radius of an electron orbiting around the ionized donor in silicon. (ϵ_r = 11.9 and μ_e^* = 0.26 μ_0)

Solution

The Bohr radius is obtained from:

$$a_o = \frac{\epsilon_o h^2 n^2}{p \, \mu_0 q^2}$$

However, since the electron travel through silicon one has to replace the permittivity of vacuum with the dielectric constant of silicon and the free electron mass with the effective mass for conductivity calculations so that:

$$a_o, donor \; in \; silicon = a_o \frac{\epsilon_r}{\mu_e^* \mu_0} = 529 \, x \, \frac{11.9}{0.26} \, pm = 2.42 \, nm$$

Example (1)

Complete Ionization of the Dopant Atoms In a silicon sample doped with 10^{17}cm^{-3} of phosphorus atoms, what fraction of the donors are not ionized (i.e., what fraction are occupied by the

"extra" electrons)?

Solution:

First, assume that all the donors are ionized and each donor donates an electron to the conduction band.

$$N = N_D = 10^{17}\ \text{cm}^{-3}$$

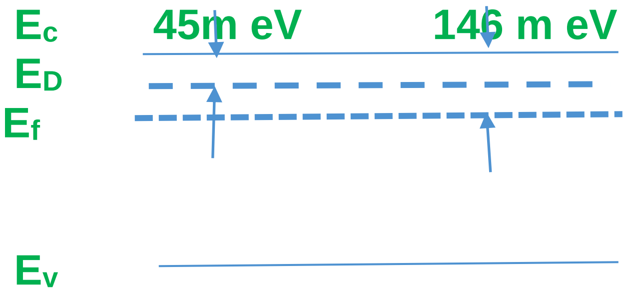
From, previous example, E_F is located at 146 meV below E_c. The donor level E_d is located at 45 meV below E_c for phosphorus (see Table).

The probability that a donor is not ionized, i.e., the probability that it is occupied by the "extra" electron, according to Eq. $f(E) = \frac{1}{1+e^{(E-E_F)/KT}}$, is :

$$\mathit{Probability\ of\ non-ionization} \approx \frac{1}{1+{}^{1}/_{2}\, e^{(E_d-E_f)/KT}}$$

$$= \frac{1}{1+{}^{1}/_{2}\, e^{(146-45)meV/26meV}} = 3.9\ \%$$

(The factor 1/2 in the denominators stems from the complication that a donor atom can hold an electron with upspin or downspin. This increases the probability that donor state occupied by an electron.) Therefore, it is reasonable to assume complete ionization, i.e., $n = N_d$.



Location of E_F and E_d. Not to scale.

Part (7) Solved PROBLEMS

Effective masses and Fermi distribution Function

Problem (1)

Consider the general exponential expression for the concentration of electrons in the CB, $n = N_c \exp -(E_c - E_F)/kT$, and the mass action law, $np = n_i^2$.

What happens when the doping level is such that n approaches N_c and exceeds it?

Can you still use the above expressions for n and p?

Consider an n-type Si that has been heavily doped and the electron concentration in the CB is 10^{20} cm^{-3}. Where is the Fermi level?

Can you use $np = n_i^2$ to find the hole concentration?

What is its resistivity?

How does this compare with a typical metal?

What use is such a semiconductor?

Solution

Consider $n = N_c \exp[-(E_c - E_F)/kT]$ (1)

and $np = n_i^2$ (2)

These expressions have been derived using the Boltzmann tail ($E > E_F$ + a few kT) to the Fermi − Dirac (FD) function f(E) as in (in the textbook).

Therefore, the expressions are NOT valid when the Fermi level is within a few kT of E_c.

In these cases, we need to consider the behavior of the FD function f(E) rather than its tail and the expressions for n and p are complicated.

It is helpful to put the 10^{20} cm^{-3} doping level into perspective by considering the number of atoms per unit volume (atomic concentration, n_{Si}) ,in the Si crystal:

$n_{at} = (\text{Density})N_A / M_{at}$

$= (2.33\times10^{3}\ \text{kg m}^{-3})(6.022\times10^{23}\ \text{mol}^{-1})\ (28.09\times10^{-3}\ \text{kg mol})$

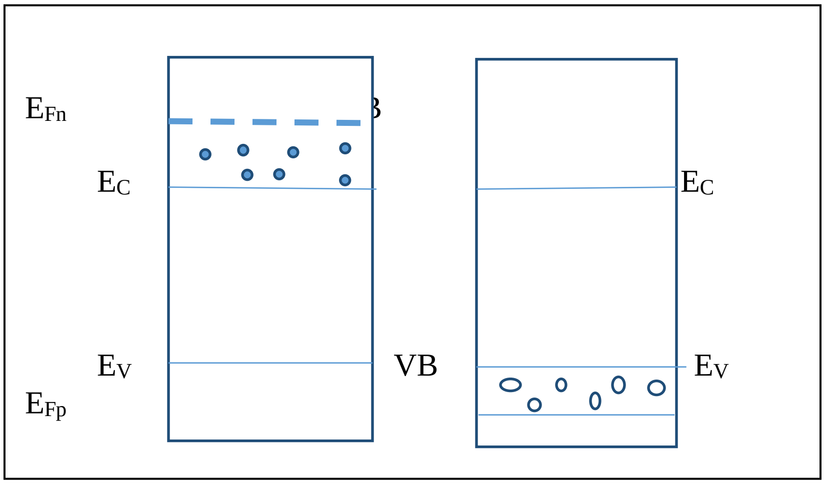
i.e.

$n_{at} = 4.995 \times 10^{28}\ \text{m}^{-3}$ or $4.995 \times 10^{22}\ \text{cm}^{-3}$

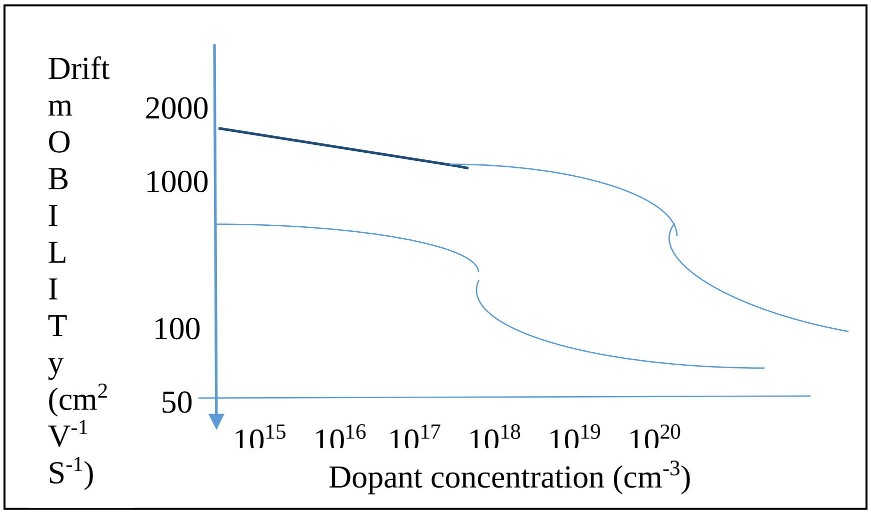
Given that the electron concentration $n = 10^{20}$ cm^{-3} (not necessarily the donor concentration!), we see that

$n/n_{at} = (10^{20}\ \text{cm}^{-3}) / (4.995 \times 10^{22}\ \text{cm}^{-3}) = 0.00200$

which means that if all donors could be ionized we would need 1 in 500 doping or 0.2% donor doping in the semiconductor (n is not exactly N_d for degenerate semiconductors). We cannot use Equation (1) to find the position of E_F. The Fermi level will be in the conduction band. The semiconductor is degenerate (see Figure 5Q7-1).



(a) Degenerate n-type semiconductor. Large number of donors form a band that overlaps the CB.
(b) Degenerate p-type semiconductor.



The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K.

Take T = 300 K, and $\mu_e \approx 900\ cm^2\ V^{-1}\ s^{-1}$ from Figure.

The resistivity is $\rho = 1/(en\ \mu_e)$

$= 1/[(1.602 \times 10^{-19}\ C)(10^{20}\ cm^{-3})(900\ cm^2\ V^{-1}\ s^{-1})]$

$\therefore\ \rho = 6.94 \times 10^{-5}\ \Omega\ cm$ or $694 \times 10^{-7}\ \Omega\ m$

Compare this with a metal alloy such as nichrome which has
$\rho = 1000\ n\Omega\ m = 10 \times 10^{-7}\ \Omega\ m$.
The difference is only about a factor of 70.
This degenerate semiconductor behaves almost like a “metal”. Heavily doped degenerate semiconductors are used in various MOS (metal- oxide- semiconductor) devices where they serve as the gate electrode (substituting for a metal) or interconnect lines.

Problem (2)

Gold in Si has two energy levels in the band gap: $E_C - E_A = 0.54$ eV, $E_D - E_F = 0.29$ eV. Assume the third level $E_D - E_F = 0.35$ eV is inactive. (a) What will be the state of charge of the gold levels in Si doped with high concentration of boron atoms? Why?
(b) What is the effect of gold on electron and hole concentrations?

Problem (3)

For an n-type silicon sample doped with $2.86 \times 10^{16}\text{cm}^{-3}$ phosphorous atoms, find the ratio of the neutral to ionized donors at 300 K. $(E_C - E_D) = 0.045$ eV.

Example (9)

Show that

$$N_D^+ = N_D \left[1 + 2\, exp\left(\frac{E_F - E_D}{KT}\right)\right]^{-1}$$

[Hint: The probability of occupancy is

$$F(E) = \left[1 + \frac{h}{g}\; exp\left(\frac{E - E_F}{KT}\right)\right]^{-1}$$

where h is the number of electrons that can physically occupy the level E, and g is the number of electrons that can be accepted by the level, also called the ground-state degeneracy of the donor impurity level (g = 2).]

Pr0blem (7)

1. **(a) Derive an expression for the total number of states in a semiconductor material (per unit volume) between E$_c$ and E$_c$ + kT, where E$_c$ is the conduction band edge (bottom of the conduction band), k is Boltzmann's constant and T is the temperature. by integrating the density of states over the energy range given in this problem.**

Solution: The density of states represents the number of states per unit volume. For this problem, we simply need to integrate the density of states, g(E)

$$g(E) = \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\sqrt{E - E_c}$$

From E$_C$ to E$_C$ + KT

$$\int_{E_c}^{E_c+KT} g_e(E)\;dE = \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{E_c+KT}(E - E_c)^{1/2}\,dE$$

$$= \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\int_{0}^{KT} x^{1/2}\;dx$$

$$= \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\frac{2}{3}(KT)^{3/2}$$

$$= \frac{1}{3\,\pi^2}\left(\frac{2\,m_e^*{}_{KT}}{h^2}\right)^{3/2}$$

$$= \left(\frac{m_e^*}{m_o}\right)^{3/2}\;x\;\frac{1}{3\,\pi^2}\left(\frac{2\,m_e^*{}_{KT}}{h^2}\right)^{3/2}$$

For values of,
m_o = 9.11 x 10^{-31} kg ,
KT = 0.026 x 1.6 x $1o^{-19}$ J,
h = 1.05 x 10^{-34} J.s

we find:

$$\int_{E_c}^{E_c+KT} g_e\,(E)\,dE = 1.97\,x\,10^{25}m^{-3} = \;1.97\,x\,10^{19}cm^{-3}$$

(b) Evaluate the expression you derived in (a) for GaAs and Si.

Solution: For GaAs, using m_e=0.067 m_0, we find 3.41×10^{17} cm^{-3}. For Si, using m_e=1.08 m_0, we find 2.21×10^{19} cm^{-3}.

Example (3)
Silicon crystal is doped with 5 x $10^{20}/m^3$ atoms per m^3 . The donor level is 0.05 eV from the edge of the conduction band. Taking the band gap to be 1.12 eV, calculate the position of the Fermi level at 200 K.

Solution
The intrinsic carrier concentration can be obtained from the known carrier concentration in Si at 300 K. As the carrier concentration at 300K is 1.5x$10^{16}/m^3$, the carrier concentration at 200 K is

$$\left(\frac{200}{300}\right)^{3/2} x\,1.5\,x\,10^{16} = 0.82\,x\,10^{16}/\mathrm{m3}$$

As the doping concentration is much larger than n_i. we can take,

$$n\;\approx\;N_D = 5\,x\,10^{20}/m^3\,,thus$$

$$E_F^n - \;E_F^i = KT\,ln\;{}^{n}/_{n_i} = 0.183\;eV$$

Exercise (4)

Germanium has ionized acceptor density of $4\,x\,10^{21}/m^3$ and donor density of $6\,x\,10^{21}/m^3$. Taking the band gap to be 0.67 eV, calculate the equilibrium density of majority and minority carriers at 450 K and also the Fermi energy.

[Hint: Using the intrinsic concentration at 300 K, find n_i at 450 K and use the expression for n]

Ans, n = $2.02 \times 10^{21}/m^3$, p = $9.62 \times 10^{17}/m^3$, $E_F^n - E_F^i = 0.143\ eV$

Problem (5)

2. Consider a silicon crystal whose band gap energy is $E_g = 1.12$ eV and whose temperature is kept at T=300◦K.

(a) If the Fermi level, E_f, is located in the middle of the band gap, what is the probability of finding an electron (or equivalently, the probability of a state being occupied) at $E = E_c + kT$.

Solution: The probability is given by the Fermi-Dirac function. Since $E_f = E_c - E_g/2$,

$$f(E) = \frac{1}{exp\left[\left(E - E_c + {}^{E_g}/_2\right)\Big/_{KT}\right] + 1}$$

As $E = E_C + KT$, we find :

$$f\,(E_c + KT) = \frac{1}{exp\left[\left(KT + {}^{E_g}/_2\right)\Big/_{KT}\right] + 1}$$

$$= \frac{1}{exp\left({}^{(0.026 + 0.56)}/_{0.026}\right) + 1} = exp\,(-22.6)$$

$$\cong 1.53\,x\,10^{-10}$$

(b) If the Fermi level, E_f is located at the conduction band edge, $E_F = E_c$, what the probability of finding an electron at E = E_c + kT.

Solution: The probability is given by evaluating the Fermi-Dirac probability density, with $E_f = E_c$

$$F(E) = \frac{1}{exp\left[{}^{(E-\ E_c)}/_{KT}\right] + 1}$$

With E = E_c + kT we find,

$$F\ (E_c + KT) = \frac{1}{exp\ (1) + 1} \cong 0.27$$

Problem (6)

The equilibrium electron concentration is given by the product of the density of states and the probability function, n(E) = g_c (E) F(E). If E −E_F>> kT, the Fermi-Dirac probability function can be approximated with the Maxwell-Boltzmann function

$$f(E) = \frac{1}{exp\left[{}^{(E-\ E_F)}/_{KT}\right] + 1} \cong exp\left[-{}^{(E-\ E_F)}/_{KT}\right]$$

(a) Using this approximation, find the energy relative to the conduction band edge, E−E_c, at which the electron concentration becomes maximum.

Solution:

The electron density, n(E) = g(E)f(E), can be written in the form

$$n\ (x) = constant\ x\sqrt{x}\ \ exp\ (-x)$$

Where x = (E−E_c)/kT, for purposes of finding the maxima. Taking the derivative and setting equal to zero

$$\frac{dn\,(x)}{dx} = constant\ x \left(\frac{1}{2\sqrt{x}} - \sqrt{x}\right) exp\,(-x) = 0$$

We find that the distribution peaks at $E - E_c = kT/2$.

(b) Using this approximation, calculate the electron concentration per unit energy interval (in units of $cm^{-3}\ eV^{-1}$) in silicon at energy $E = E_c - kT$. Assume the Fermi level is located at the center of the band gap, $E_F = E_c - E_g/2$.

Solution: We want to evaluate

$$n\,(E) = g(E)f(E)$$

$$= \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\sqrt{(E - E_c)}exp\left[-{}^{(E - E_F)}/_{KT}\right]$$

at $E = E_c + kT$ for $E_f = \varepsilon_g/2$.

The result is $n(E_c + kT) = g(E)f(E)$

$$n\,(E) = g(E)f(E)$$

$$= \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\sqrt{KT}\,exp\left[-{}^{\left(KT + {}^{E_g}/_2\right)}/_{KT}\right]$$

$$n\,(E) = g(E)f(E)$$

$$= \frac{1}{2\,\pi^2}\left(\frac{2\,m_e^*}{h^2}\right)^{3/2}\sqrt{0.026}\,exp\left[-{}^{(0.026 + 0.56)}/_{0.026}\right]$$

$$= 1.84\ x\ 10^{11}\ cm^{-3}eV^{-1}$$

Repeat the calculation in (b) without using the approximation.

Solution: The answer is really close, to within a part in roughly 10^{-10}.

Problem (7)

If a silicon sample is doped with 10^{16} phosphorous

impurities/cm^3, find the ionized donor density at 77 K. Assume that the ionization energy for phosphorous donor impurities and the electron effective mass are independent of temperature. (Hint: First select a N^+_D value to calculate the Fermi level, then find the corresponding N^+_D . If they don't agree, select another N^+_D value and repeat the process until a consistent N^+_D is obtained.)

Problem (8)

Using graphic method to determine the Fermi level for a boron-doped silicon sample with an impurity concentration of 10^{15} cm^{-3} at 300 K (note $n_i = 9.65\text{x}10^4$ crn^{-3}).

Problem (9)

The Fermi-Dirac distribution function is given by

$$\boldsymbol{f\ (E) = \frac{1}{1 + exp\ ^{(E -\ E_F)}/_{KT}}}$$

The differentiation of F(E) with respect to energy is F'(E). Find the width of F'(E), i.e., $2\left[E\,(at\,F_{max}) - E\,\left(at\frac{1}{2}F_{max}\right)\right]$, where F_{max} is the maximum value of F'(E).

Problem (10)

Find the position of the Fermi level with respect to the bottom of the conduction band (E_c– E_F) for a silicon sample at 300 K, which is doped with 2x 10^{10}cm^{-3} fully ionized donors.

Problem (11)

The Gamma Function is defined as

$$\boldsymbol{\Gamma(n) = \int_0^n x^{n-1}\ exp\,(-x)\,dx}$$

(a) Find Γ(1/2), and (b) show that Γ(n) = (n - I)Γ(n - I).

Example 1.5

Calculate the energy relative to the Fermi energy for which the Fermi function equals 5%. Write the answer in units of kT.

Solution:

The problems states that

$$f\,(E) = \frac{1}{1 + exp\,\left(\frac{E - E_F}{KT}\right)} = 0.05$$

Which can be solved yielding?

$$E - E_F = ln\,(19)\,KT = 3\,KT$$

Problem 1.9

Prove that the probability of occupying an energy level below the Fermi energy equals the probability that an energy level above the Fermi energy and equally far away from the Fermi energy is not occupied.

Solution

The probability that an energy level with energy ∆E below the Fermi energy E_F is occupied can be rewritten as:

$$\begin{aligned} f(E_F - \Delta\,E) &= \frac{1}{1 + exp\,\frac{E_F - \Delta E - E_F}{KT}} = \frac{exp\,\frac{\Delta E}{KT}}{exp\,\frac{\Delta E}{KT} + 1} \\ &= 1 - \frac{1}{exp\,\frac{\Delta E}{KT} + 1} = 1 - \frac{1}{1 + exp\,\frac{E_F + \Delta E - E_F}{KT}} \\ &= 1 - f(E_F + \Delta\,E) \end{aligned}$$

so that it also equals the probability that an energy level with energy ∆E above the Fermi energy, E_F, is not occupied.

Problem 2.3

Prove that the probability of occupying an energy level below the Fermi energy equals the probability that an energy level above the Fermi energy and equally far away from the Fermi energy is not occupied.

Solution

The probability that an energy level with energy ∆E below the Fermi energy E_F is occupied can be rewritten as

$$f\ (E_F\ \Delta E) = \frac{1}{1 + exp\ \frac{E_F - \Delta E - E_F}{KT}} = \frac{exp\ \frac{\Delta E}{KT}}{exp\ \frac{\Delta E}{KT} + 1}$$

$$= 1 - \frac{1}{exp\ \frac{\Delta E}{KT} + 1} = 1 - \frac{1}{1 + exp\ \frac{E_F - \Delta E - E_F}{KT}} = 1 - f\ (E_F + \ \Delta\ E)$$

so that it also equals the probability that an energy level with energy ∆E above the Fermi energy, E_F, is not occupied.

Problem 2.4 At what energy (in units of kT) is the Fermi function within 1 % of the Maxwell-Boltzmann distribution function? What is the corresponding probability of occupancy?

Solution

The Fermi function can be approximated by the MaxwellBoltzmann distribution function with an approximate error of 1 % if

$$\frac{f_{mB} - \ f_{FB}}{f_{FD}} = 0.01\ or\ \frac{1}{f_{FD}} = \frac{1.01}{f_{mB}}$$

using x = (E - E_F)/kT,

this condition can be rewritten as:

1 + exp (x) = 1.01 exp(x)

from which one finds

x = ln(100) = 4.605

so that

E = E_F + 4.605 kT

and $f_{FD}(E_F + 4.605\ kT) = 0.0099$

Problem 2.5

Calculate the Fermi function at 6.5 eV if E_F = 6.25 eV and T = 300 K. Repeat at T = 950 K assuming that the Fermi energy does not change. At what temperature does the probability that an energy level at E = 5.95 eV is empty equal 1 %.

Solution The Fermi function at 300 K equals:

$$f(6.25\ eV) = \frac{1}{1 + exp\left(\frac{6.5-6.25}{0.0258}\right)} = 6.29\ x\ 10^{-5}$$

The Fermi function at 950 K equals:

$$f(6.25\ eV) = \frac{1}{1 + exp\left(\frac{6.5-6.25}{0.0818}\right)} = 0.045$$

The probability that the Fermi function equals 1 % implies:

$$f(5.95\ eV) = 0.99 = \frac{1}{1 + exp\left(\frac{5.95-6.25}{KT/q}\right)}$$

resulting in

$$T = \frac{0.3\ \frac{q}{K}}{ln\left(\frac{1}{0.99} - 1\right)} = 484.7\ o_C$$

Example 2.7

A piece of germanium doped with 10^{16} cm^{-3} shallow donors is illuminated with light generating 10^{15} cm^{-3} excess electrons

and holes. Calculate the quasi-Fermi energies relative to the intrinsic energy and compare it to the Fermi energy in the absence of illumination.

Solution

The carrier densities when illuminating the semiconductor are: -

$$n = n_o + dn = 10^{16} + 10^{15} = 1.1\ x\ 10^{16}\ cm^{-3}$$

$$p = p_o + dp = 10^{16}\ cm^{-3}$$

and the quasi-Fermi energies are:

$$F_n - E_i = KT\ ln\ \frac{n}{n_i} = 0.0259\ x\ ln\ \frac{11.1\ x\ 10^{16}}{2\ x\ 10^{13}} = 163\ meV$$

$$F_p - E_i = KT\ ln\ \frac{n}{n_i} = 0.0259\ x\ ln\ \frac{1\ x\ 10^{15}}{2\ x\ 10^{13}} = 101\ meV$$

In comparison, the Fermi energy in the absence of light equals

$$F_F - E_i = KT\ ln\ \frac{n_o}{n_i} = 0.0259\ x\ ln\ \frac{10^{16}}{2\ x\ 10^{13}} = 161\ meV$$

which is very close to the quasi-Fermi energy of the majority carriers.

Problem 2.8

Calculate the position of the intrinsic energy level relative to the midgap energy $E_{mid\ gap} = (E_c + E_v)/2$ in germanium, silicon and gallium arsenide at 300 K. Repeat at T = 100 °C.

Solution:

The intrinsic energy level relative to the midgap energy is obtained from:

$$E_i = E_{mid\ gap} = \frac{3}{4}\ KT\ ln\ \frac{m_h^*}{m_e^*}$$

where the effective masses are the effective masses for density of states calculations as listed in the table below. The

corresponding values of the intrinsic level relative to the midgap energy are listed as well.

45.92 meV	Germanium	Silicon	Gallium arsenide
m_e^*/m_0	0.55	1.08	0.067
m_h^*/m_0	0.37	0.81	0.45
T = 300 K	7.68 meV	5.58 meV	36.91 meV
T = 100 C	9.56 meV	6.94 meV	45.92 meV

Problem 2.9

Calculate the electron and hole density in germanium, silicon and gallium arsenide if the Fermi energy is 0.3 eV above the intrinsic energy level. Repeat if the Fermi energy is 0.3 eV below the conduction band edge. Assume that T = 300 K.

Solution

The electron density, n, can be calculated from the Fermi energy using:

$$n = n_i \, exp \, \frac{E_F - E_i}{KT} = n_i \, exp \left(\frac{0.3}{0.0258}\right)$$

and the corresponding hole density equals

$$p = \frac{n_i^2}{n}$$

the resulting values are listed in the table below.

If the Fermi energy is 0.3 eV below the conduction band edge, one obtains the carrier densities using:

$$n = N_c \, exp \, \frac{E_F - E_c}{KT} = N_c \, exp \left(\frac{0.3}{0.0258}\right)$$

and the corresponding hole density equals:

$$p = \frac{n_i^2}{n}$$

the resulting values are listed in the table below.

		Germanium	Silicon	Gallium Arsenide

	ni (cm-3)	**2.03 x 1013**	**1.45 x 1010**	**2.03 x 106**
	Nc (cm-3)	**1.02 x 1019**	**6.62 x 1019**	**4.37 x 1017**
EF - Ei = 0.3 eV	**n (cm-**	**2.24 x 1018**	**1.60 x 1015**	**2.23 x 1011**
	p (cm-3)	1.48 x 108	1.32 x 105	18.4
EF - Ei = - 0.3 eV	**n (cm-3)**	**9.27 x 1013**	**6.02 x 1014**	**3.97 x 1012**
	p (cm-3)	4.45 x 1012	3.50 x 105	1.04

Problem 2.10

The equations

$$n_o = \frac{N_d^+ - N_a^-}{2} + \sqrt{\left(\frac{N_d^+ - N_a^-}{2}\right)^2 + n_i^2}$$

$$p_o = \frac{N_a^- - N_d^+}{2} + \sqrt{\left(\frac{N_a^- - N_d^+}{2}\right)^2 + n_i^2}$$

are only valid for non-degenerate semiconductors (i.e. E_v + 3kT < E_F<E_c - 3kT). Where exactly in the derivation was the assumption made that the semiconductor is non-degenerate?

Solution

The above two Equations were derived using charge neutrality and the mass action law. Of those two assumptions, the use of the mass action law implies that the semiconductor is nondegenerate.

The mass action law was

$$n_o \cong \int_{E_c}^{\infty} \frac{8\,\pi\,\sqrt{2}}{h^3}\,\mu_e^{*\,3/2}\sqrt{E - E_c}\,e^{\frac{(E_F - E)}{KT}}\,dE = N_c e^{\frac{(E_F - E_c)}{KT}}$$

$$N_c = 2\left(\frac{2\,\pi\mu_e^*\,KT}{h^2}\right)^{3/2}$$

These equations, representing a closed form solution for the thermal equilibrium carrier densities as a function of the

Fermi energy, were in turn obtained by solving the Fermi integral and assuming that:

$$E_v + 3kT < E_F < E_c - 3kT$$

i.e. that the Fermi energy must be at least 3kT away from either bandedge and within the bandgap.

Problem 2.31

Find the equilibrium electron and hole concentrations and the location of the Fermi energy relative to the intrinsic energy in silicon at 27 °C, if the silicon contains the following concentrations of shallow dopants.

a) 1 x 10^{16} cm^{-3} boron atoms

b) 3 x 10^{16} cm^{-3} arsenic atoms and 2.9 x 10^{16} cm^{-3} boron atoms.

Solution

a) Boron atoms are acceptors, therefore $N_a = 10^{16}$ cm^{-3} Since these are shallow acceptors and the material is not compensated, degenerate or close to intrinsic, the hole density equals the acceptor density: $p \approx 10^{16}$ cm^{-3} Using the mass action law we then find the electron density $n = n^2i/p$ = 1 x 10^4 cm^{-3} The Fermi energy is then obtained from:

$$E_F - E_i = KT\, ln\, \frac{n}{n_i} = 0.0259\, ln\, \frac{10^4}{10^{10}} = -375\, meV$$

b)

Arsenic atoms are donors, therefore N_a = 2.9 x 10^{16} cm^{-3} and N_d = 3 x 10^{16} cm^{-3} Since these are shallow acceptors and the material is not degenerate or close to intrinsic, the electron density approximately equals the difference between the donor and acceptor density $n \approx N_d - N_a = 10^{15}$ cm^{-3} Using the mass action law we then find the hole density $p = n^2_i/n$ = 1 x 10^5 cm^{-3} The Fermi energy is then obtained from:

$$E_F - E_i = KT\, ln\, \frac{n}{n_i} = 0.0259\, ln\, \frac{10^{15}}{10^{10}} = 298\, meV$$

Example (2) 1–2

Oxygen Concentration versus Altitude , We all know that there is less oxygen in the air at higher altitudes. What is the ratio of the oxygen concentration at 10 km above sea level, N_h, to the concentration at sea level, N0, assuming a constant temperature of 0°C?

SOLUTION: There are fewer oxygen molecules at higher altitudes because the gravitational potential energy of an oxygen molecule at the higher altitude, E_h, is larger than at sea level, E_0. According to Equation

$$f(E) \approx e^{-(E - E_F)/_{KT}}$$

$$\frac{N_h}{N_0} = \frac{e^{-E_g/_{KT}}}{e^{-E_0/_{KT}}} = e^{-\left(E_h - E_0/_{KT}\right)}$$

E_0 –E_h is the potential energy difference, i.e., the energy needed to lift an oxygen molecule from sea level to 10 km.

E_h E_0= altitude x weight of O_2 molecule x acceleration of gravity

= 10^4 m x O_2 molecular weight atomic mass unit x 9.8 m s^{-2}

= 10^4 m x 32 x1.66 x10^{27} kgx 9.8 m s^{-2}= 5.2 x 10^{-21} J

$$\therefore \frac{N_h}{N_0} = e^{-5.2\,x\,10^{21}/_{1.38\,x\,10^{-10}}\,J.K^{-1}\,x\,273\,K} = e^{-1.38} = 0.25$$

Therefore, the oxygen concentration at 10 km is 25% of the sea level concentration. This example and the sand-in-a-dish analogy presented to demystify the concept of equilibrium, and to emphasize that each electron energy state has a probability occupied that governed by the Fermi function.

Example (3)

Finding the Fermi Level in Si Where is E_F located in the energy band of silicon, at 300K with n = $10^{17}$$cm^{-3}$? In addition, for p =

10^{14}cm^{-3}?

Solution:

From Eq.

$$n = N_c e^{-(E_c - E_F)/KT}$$

$$E_c - E_f = KT \ln \left({N_c}/{n} \right)$$

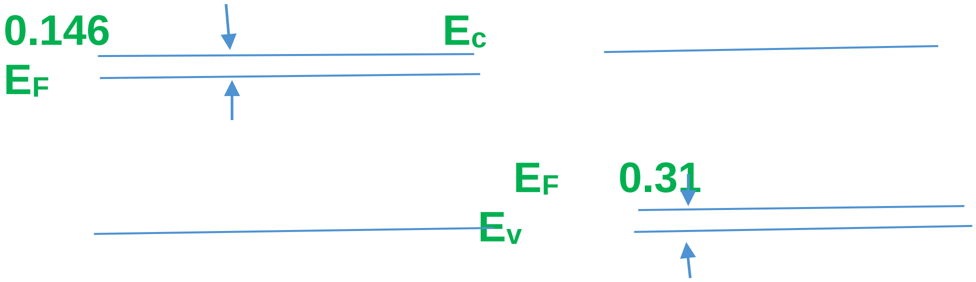
$$= 0.026 \ln \left({2.8 \, x \, 10^{19}}/{10^{17}} \right) = 0.146 \, eV$$

Therefore, E_F is located at 146 meV below E_c, as shown in Figure. For p = 10^{14}cm^{-3}, from Equation,

$$E_F - E_v = KT \ln \left({N_v}/{p} \right)$$

$$= 0.026 \ln \left({1.4 \, x \, 10^{19}}/{10^{14}} \right) = 0.31 \, eV$$

Therefore, E_F is located at 0.31 eV above E_v.



Example (1)

Complete Ionization of the Dopant Atoms In a silicon sample doped with 10^{17}cm^{-3} of phosphorus atoms, what fraction of the donors are not ionized (i.e., what fraction are occupied by the "extra" electrons)?

Solution:

First, assume that all the donors are ionized and each donor donates an electron to the conduction band.

$$N = N_D = 10^{17} \text{ cm}^{-3}$$

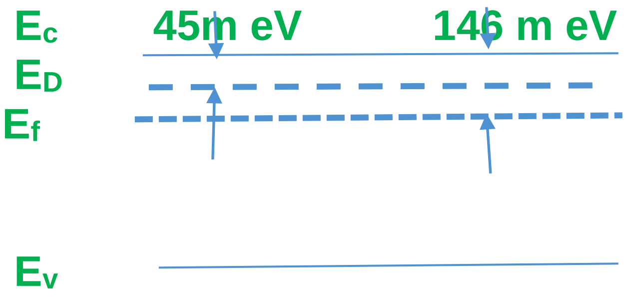
From, previous example, E_F is located at 146 meV below E_c. The donor level E_d is located at 45 meV below E_c for phosphorus (see Table).

The probability that a donor is not ionized, i.e., the probability that it is occupied by the "extra" electron, according to Eq. $f(E) = \frac{1}{1+ e^{(E- E_F)}/_{KT}}$, is :

$$Probability\ of\ non-ionization\ \approx \frac{1}{1+\ {}^{1}/_{2}\, e^{(E_d - E_f)}/_{KT}}$$

$$= \frac{1}{1+\ {}^{1}/_{2}\, e^{(146-45)meV}/_{26meV}} = 3.9\ \%$$

(The factor 1/2 in the denominators stems from the complication that a donor atom can hold an electron with upspin or downspin. This increases the probability that donor state occupied by an electron.) Therefore, it is reasonable to assume complete ionization, i.e., n = N$_d$.



Location of E$_F$ and E$_d$. Not to scale.

Part (8) Solved PROBLEMS

Energy: Density of States

Example 2.3

Calculate the number of states per unit energy in a 100 by 100 by 10 nm piece of silicon (m* = 1.08 m$_0$) 100 meV above the conduction band edge. Write the result in units of eV-1.

Solution

The density of states equals:

$$g(E) = \frac{8\,p\,\sqrt{2}}{h^3} m^{*^{3}/_{2}} \sqrt{E - E_c}$$

$$= \frac{8\,p\,\sqrt{2}(1.08\,x\,9.1x\,10^{-31})^{3/_{2}}}{(6.626\,x\,10^{-34})^2} \sqrt{0.1\,x\,1.6\,x\,10^{-19}}$$

$$= 1.51\,x\,10^{56} m^{-3} J^{-1}$$

So that the total number of states per unit energy equals

$$g\,(E)\,V = 1.51\,x\,10^{56}\,x\,10^{-22} J^{-1} = 2.41\,x\,10^{5} eV^{-1}$$

Example (7)

Derive the density of states in the conduction band as given by Eq.

$$N\,(E) M_c \frac{\sqrt{2}}{\pi^2} \frac{m_{de}^{3/_{2}} (E - E_c)^{1/_{2}}}{h^3}$$

(Hint: The wavelength ٨ of a standing wave is related to the length of the semiconductor L by v٨ = n_x where n_x is an integer. The wavelength can be expressed by de Broglie hypothesis ٨ = h/p_x. Consider a three-dimensional cube of side L)

Example (8)

10. Calculate the average kinetic energy of electrons in the conduction band of an n-type non- degenerate semiconductor. The density of states is given by Eq.

$$N\,(E) M_c \frac{\sqrt{2}}{\pi^2} \frac{m_{de}^{3/_{2}} (E - E_c)^{1/_{2}}}{h^3}$$

Exercise (1)

Derive the expression.

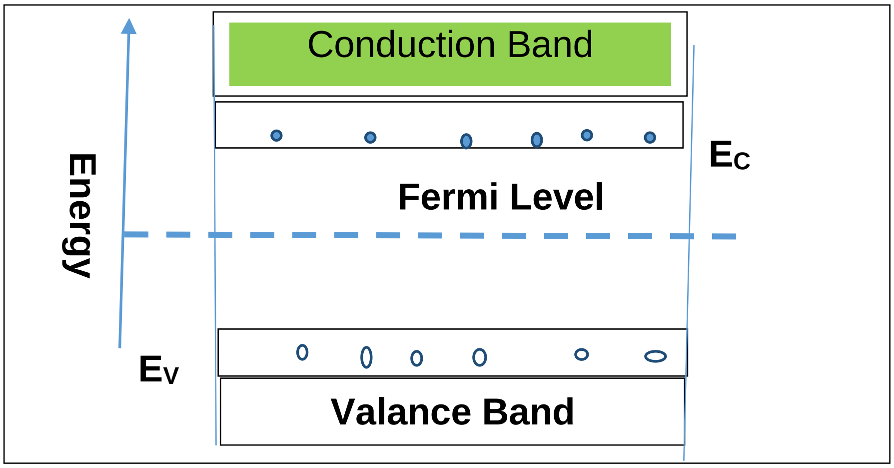
$$p = \frac{1}{4}\left(\frac{2\,m_h\,KT}{\pi\,h^2}\right)^{3/2} e^{(E_v - E_F)/KT} = N_v\,e^{(E_v - E_{cF})/KT}$$

For an intrinsic semiconductor the number of electrons in the conduction band is equal to the number of holes in the valence band since a hole is left in the valence band only when an electron makes a transition to the conduction band,

$$n = p$$

Using this and assuming that the effective masses of the electrons and holes are the same one gets,

$$e^{(E_F - E_c)/kT} = e^{(E_v - E_F)/kT}$$



Giving :

$$E_F = \frac{E_C + E_V}{2} \qquad (C)$$

i.e. the Fermi level lies in the middle of the forbidden gap. Note that there is no contradiction with the fact that no state exists in the gap as is only an energy level and not a state. By substituting the above expression for Fermi energy in(A) or (B),

$$n = \frac{1}{4}\left(\frac{2\,m_e\,KT}{\pi\,h^2}\right)^{3/2} e^{(E_F - E_C)/KT} = N_C\,e^{(E_F - E_c)/KT} \qquad (A)$$

$$p = \frac{1}{4}\left(\frac{2\,m_h\,KT}{\pi\,h^2}\right)^{3/2} e^{(E_v - E_F)/KT} = N_v\,e^{(E_v - E_{cF})/KT} \qquad (B)$$

We obtain an expression for the number density of electrons or holes (n = p = n$_i$)

$$n_i = \frac{1}{4}\left(\frac{2\,KT}{\pi\,h^2}\right)^{3/2} (m_e m_h)^{3/4}\,e^{-\Delta/2\,KT} \qquad (D)$$

Where Δ is the width of the gap

Example 2.4

Calculate the effective densities of states in the conduction and valence bands of germanium, silicon and gallium arsenide at 300 K.

Solution

The effective density of states in the conduction band of germanium equals:

$$N_c = 2\left(\frac{2\,p\,m_e^*\,KT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\,p\,x\,0.55\,x\,9.11\,x\,10^{-31}\,x\,1.38\,x\,10^{-22}\,x\,300}{(6.626\,x\,10^{-34})^2}\right)^{3/2}$$

$$= 1.02\,x\,10^{25} m^{-3} = 1.02\,x\,10^{19} cm^{-3}$$

where the effective mass for density of states was. Similarly, one finds the effective density of states in the conduction band for other semiconductors and the effective density of states in the valence band:

	Germanium	Silicon	Gallium Arsenide
Nc (cm^{-3})	1.02 x 10^{19}	2.81 x 10^{19}	4.35 x 10^{17}
Nv (cm^{-3})	5.64 x 10^{18}	1.83 x 10^{19}	7.57 x 10^{18}

Note that the effective density of states is temperature dependent and can be obtain from:

$$N_c(T) = N_c(300K)\left(\frac{T}{300}\right)^{3/2}$$

where N_c(300 K) is the effective density of states at 300 K.

Part (9) Solved PROBLEMS
Compounds

Problem (1)

Consider the GaAs crystal at 300 K.

a. Calculate the intrinsic conductivity and resistivity.

b. In a sample containing only 10^{15} cm^{-3} ionized donors, where is the Fermi level? What is the conductivity of the sample?

c. In a sample containing 10^{15} cm^{-3} ionized donors and 9 × 10^{14} cm^{-3} ionized acceptors, what is the free hole concentration?

Solution:

a - Given temperature, T = 300 K, and intrinsic GaAs.

From Tables,

$n_i = 1.8 \times 10^6$ cm^{-3}, $\mu_e \approx 8500$ cm^2 V^{-1} s^{-1} and $\mu_h \approx 400$ cm^2 V^{-1} s^{-1}.

Thus, $\sigma = en_i(\mu_e + \mu_h)$

$\therefore \sigma = (1.602 \times 10^{-19}$ C$)(1.8 \times 10^6$ cm$^{-3})(8500$ cm^2 V^{-1} s^{-1} + 400 cm^2 V^{-1} s$^{-1})$

$\therefore \sigma = 2.57 \times 10^{-9} \Omega^{-1}$ cm^{-1}

$\therefore \rho = 1/\sigma = 3.89 \times 10^8 \Omega$ cm

B - Donors are now introduced.

At room temperature, $n = N_d = 10^{15}$ cm$^{-3} >> n_i >> p$.

$\sigma n = eN_d\mu_e \approx (1.602 \times 10^{-19}$ C$)(10^{15}$ cm$^{-3})(8500$ cm^2 V^{-1} s$^{-1}) = 1.36\ \Omega^{-1}$ cm^{-1} $\therefore \rho_n = 1/\sigma_n = 0.735\ \Omega$ cm

In the intrinsic sample,

$E_F = E_{Fi}$, $n_i = N_c\exp[-(E_c - E_{Fi})/kT]$ (1)

In the doped sample,

$n = N_d$, $E_F = E=$, $n = N_d = N_c \exp[-(E_c - E_{Fn})/kT]$ (2)
Eqn. (2) divided by Eqn. (1) gives,
$N_d/n_i = \exp (E_{Fn} - E_{Fi})/ kT$ (3)
$\therefore \Delta E_F = E_{Fn} - E_{Fi} = kT \ln(N_d/n_i)$ (4)
Substituting we find,
$\Delta E_F = (8.617 \times 10^{-5}$ eV/K)(300 K)$\ln[(10^{15}$ cm$^{-3})/(1.8 \times 10^6$ cm$^{-3})]$
$\therefore \Delta E_F = 0.521$ eV above E_{Fi} (intrinsic Fermi level)

c The sample is further doped with $N_a = 9 \times 10^{14}$ cm^{-3} = 0.9×10^{15} cm^{-3} acceptors. Due to compensation, the net effect is still an n-type semiconductor but with an electron concentration given by,
$n = N_d - N_a = 10^{15}$ cm^{-3} − 0.9×10^{15} cm^{-3} = 1×10^{14} cm^{-3} ($>>n_i$)
The sample is still n-type though there are less electrons than before due to the compensation effect. From the mass action law, the hole concentration is:
$p = n_i^2 / n = (1.8 \times 10^6$ cm$^{-3})^2 / (1 \times 10^{14}$ cm^{-3}) = 0.0324 cm^{-3}
On average there are virtually no holes in 1 cm^3 of sample. We can also calculate the new conductivity. We note that electron scattering now occurs from $N_a + N_d$ number of ionized centers though we will assume that
$\mu_e \approx 8500$ cm^2 V^{-1} s^{-1}.
$\sigma = en\mu_e \approx (1.602 \times 10^{-19}$ C)(10^{14} cm^{-3})(8500 cm^2 V^{-1} s^{-1}) = 0.136 Ω^{-1} cm^{-1}

Problem (2)
AlN and GaSb are compounds, solid at room temperature. On the basis of bonding considerations and data provided in the periodic table, attempt to predict differences in the properties of these solids.
Solution
Both compounds are of the III–V family, which hybridize and form "adamantine" (diamond–like) structures which places them into the category of semiconductor.
AlN ΔEN = 1.43. The covalent radii of the constituents are small and, combined with the large EN, the bonds (polar covalence's) are very strong – the semiconductor is expected to exhibit a large band gap (likely transparent).
GaSb ΔEN = 0.24. The covalent radii of both constituents are significantly larger (than those of AlN), the ionic contribution to bonding is small – the semiconductor is expected to exhibit a much smaller band gap than AlN.
AlN: $E_g = 3.8$ eV ,GaSb: $E_g = 0.8$ eV

Problem (4)

Calculate thermal velocity of electrons and holes in GaAs at room temperature. Effective masses are $m^*_e/m_0 = 0.063$ and $m^*_h/m_0 = 0.53$.

Solution:

$$v_t = \frac{\int_0^{\infty} v \, exp\left(-{m^* v^2}/{2KT}\right) d^3\, v}{\int_0^{\infty} exp\left(-{m^* v^2}/{2KT}\right) d^3\, v} = \sqrt{\frac{8KT}{\pi\, m^*}}$$

Thermal velocities of electrons and holes are 4.3×10^7 and 1.5×10^7 cm/s, respectively.

Part (10) Solved PROBLEMS
Transparent Semiconductors

1. Which of the following semiconductors are transparent, partially transparent, nontransparent for visible light (λ = 0.4 – 0.7 μm): Si, GaAs, GaP, and GaN?

Solution: (1.1)

1. It follows from Table 2 that Si and GaAs are not transparent, GaP is partially transparent, and GaN is transparent for the visible light.

Problem (3)

1. Which of the following semiconductors are transparent, partially transparent, non transparent for visible light (λ = 0.4–0.7 μm): Si, GaAs, GaP, and GaN?

Solution:

1. It follows from Table 2 that Si and GaAs are not transparent, GaP is partially transparent, and GaN is transparent for the visible light.

Example (1) Measuring the Band-Gap Energy if a semiconductor is transparent to light with a wavelength longer than 0.87 µm, what is its band-gap energy?

SOLUTION: Photon energy of light with 0.87 µm wavelength is, with c being the speed of light

$$h\,v = h\,\frac{c}{\lambda} = \frac{6.63\ x\ 10^{-34}(j.s)\ x\ 3\ x\ 10^{8}\ m/s}{0.87\ \mu m}$$

$$= \frac{1.99\ x\ 10^{-19}\ j\ \mu m}{o.87\ \mu m}$$

$$= \frac{1.99\ x\ 10^{-19}\ eV\ \mu m}{1.6\ x\ 10^{-19}\ x\ o.87\ \mu m}$$

$$= \frac{1.24\ eV\ \mu m}{o.87\ \mu m} = 1.42\ eV$$

Therefore, the band gap of the semiconductor is 1.42 eV. The semiconductor is perhaps GaAs (Table).

Band-gap energies of selected semiconductors.

Semiconductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
Eg (eV)	0.18	0.67	1.12	1.42	2.25	2.7	6.0

Part (11) Solved PROBLEMS
THE DRIFT CURRENT , THE DIFFUSION CURRENT

Example 1.5
Calculate the built-in potential barrier of a pn junction. Consider a silicon pn junction at T = 300 K, doped at $N_a = 10^{16}$ cm^{-3} in the p-region and $N_d = 10^{17}$cm^{-3} in the n-region.

Solution:
We have $n_i = 1.5 \times 10^{10}$cm^{-3} for silicon at room temperature. We then find

$$V_{bi} = V_T \ln\left(\frac{N_d n_A}{n_i^2}\right) = 0.026 \ln\left[\frac{10^{16} x\, 10^{17}}{(1.5\, x\, 10^{10})^2}\right] = 0.757\ V$$

Comment: Because of the log function, the magnitude of V_{bi} is not a strong function of the doping concentrations. Therefore, the value of V_{bi} for silicon pn junctions is usually within 0.1 to 0.2 V of this calculated value.

Exercise 1.5
Calculate V_{bi} for a GaAs pn junction at T = 300 K for $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{17}$ cm^{-3} (b) Repeat part (a) for a Germanium pn junction with the same doping concentrations.

Solution: (a) V_{bi} = 1.23 V, (b) V_{bi} = 0.374 V).

Example 2.10
The hole density in an n-type silicon wafer ($N_d = 10^{17}$ cm^{-3}) decreases linearly from 10^{14} cm^{-3} to 10^{13} cm^{-3} between x = 0 and x = 1 µm. Calculate the hole diffusion current density.

Solution
The hole diffusion current density equals:

$$J_p = q\, D_p \frac{dp}{dx} = 1.6\, x\, 10^{-19}\, x\, 8.2\, x\, \frac{9\, x\, 10^{13}}{10^{-4}} = 1.18\, A/m^2$$

$$D_p = V_t m_p = 0.0259\ x\ 317 = 8.2\ {}^{m^2}/_{s}$$

and the hole mobility in the n-type wafer was obtained from Table as the hole mobility in a p-type material with the same doping density.

Problem 2.18

Consider the problem of finding the doping density, which results in the maximum possible resistivity of silicon at room temperature. ($n_i = 10^{10}$ cm^{-3}, $\mu_n = 1400$ cm^2/V-sec and μp = 450 cm^2/V-sec.) Should the silicon be doped at all or do you expect the maximum resistivity when dopants are added? If the silicon should be doped, should it be doped with acceptors or donors (assume that all dopant is shallow). Calculate the maximum resistivity, the corresponding electron and hole density and the doping density.

Solution

Since the mobility of electrons is larger than that of holes, one expects the resistivity to initially decrease as acceptors are added to intrinsic silicon. The maximum resistivity (or minimum conductivity) is obtained from:

$$\frac{ds}{dn} = q\ \frac{d\left(n\,\mu_n + p\,\mu_p\right)}{dn} = q\ \frac{d\ \left(n\,\mu_n + \frac{n_i^2}{n}\mu_p\right)}{dn} = 0$$

which yields:

$$n = \sqrt{\frac{\mu_p}{\mu_n}}\,n_i = 0.57\,n_i = 5.7\ x\ 10^9 cm^{-3}$$

The corresponding hole density equals p = 1.76 n_i = 1.76 x 10^9 cm^{-3} and the amount of acceptors one needs to add equals N_a = 1.20 n_i = 1.20 x 10^9 cm^{-3}. The maximum resistivity equals:

$$\rho_{max} = \frac{1}{q\left(n\,\mu_n + p\,\mu_p\right)} = \frac{1}{q\,n_i\ x\ 1587} = 394\ K\Omega\ cm$$

Problem 2.30

Phosphorous donor atoms with a concentration of 10^{16} cm^{-3} are added to a piece of silicon. Assume that the phosphorous atoms are distributed homogeneously throughout the silicon. The atomic weight of phosphorous is 31.

a) What is the sample resistivity at 300 K?

b) What proportion by weight does the donor impurity comprise? The density of silicon is 2.33 gram/cm^3.

c) If 10^{17} atoms cm^{-3} of boron are included in addition to phosphorous, and distributed uniformly, what is the resulting resistivity and type (i.e., p- or n-type material)?

d) Sketch the energy-band diagram under the condition of part c) and show the position of the Fermi energy relative to the valence band edge.

Solution

a) The electron mobility in the silicon equals

$$\mu_n = \mu_{max} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N_D}{N_\rho}\right)^a} = 68.5 + \frac{1414 - 68.5}{1 + \left(\frac{10^{16}}{9.2 \ x \ 10^{16}}\right)^{0.711}}$$

$$= 1184 \ cm^2/vs$$

$$\rho = \frac{1}{s} = \frac{1}{q\left(n\,\mu_n + p\,\mu_p\right)} = \frac{1}{1.6 \ x \ 10^{-19} \ x \ 1184 x \ 10^{16}}$$

$$= 0.53 \ \Omega \ cm$$

b)

$$\frac{\left.\frac{weight}{volume}\right|_{p}}{\left|\left.\frac{weight}{volume}\right|_{p+si}\right|} = \frac{M_m \, A \, N_d}{density \; of \; si} = \frac{31 \, x \, 1.6 \, x \, 10^{27} \, x \, 10^{3} \, x \, 10^{16}}{2928}$$

$$= 2.1 \, x \, 10^{-7}$$

c)

The semiconductor is p-type since $N_a > N_d$ The hole density is obtained from:

$$p = \frac{N_a^{+} - N_d^{-}}{2} + \sqrt{\left(\frac{N_a^{+} - N_d^{-}}{2}\right)^2 + n_i^2}$$

$$= \frac{9 \, x \, 10^{16}}{2} + \sqrt{\left(\frac{9 \, x \, 10^{16}}{2}\right)^2 + (10^{10})^2} = 9 \, x \, 10^{16} cm^{-3}$$

and the mobility is calculated from the sum of the donor and acceptor densities

$$\mu_p = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + \left(\frac{N}{N_\rho}\right)^{a}} = 44.9 + \frac{470.5 - 44.9}{1 + \left(\frac{11x \, 10^{16}}{2.23 \, x \, 10^{17}}\right)^{0.711}}$$

$$= 310.6 \, cm^2/vs$$

leading to the conductivity of the material:

$$\rho = \frac{1}{s} = \frac{1}{q \left(n \, \mu_n + p \, \mu_p\right)} = \frac{1}{1.6 \, x \, 10^{-19} \, x \, 310.6x \, 9x \, 10^{16}}$$

$$= 0.22 \, \Omega \, cm$$

d)

$$E_F - E_v = KT \, ln \, \frac{N_v}{p} = 0.0259 \, ln \, \frac{10^{14}}{9 \, x \, 10^{16}} = 123 \, meV$$

Problem 2.32

The electron concentration in a piece of lightly doped, n-type silicon at room temperature varies linearly from 10^{17} cm^{-3} at x = 0 to 6 x 10^{16} cm^{-3} at x = 2 µm. Electrons are supplied to keep this concentration constant with time. Calculate the electron current density in the silicon if no electric field is present.

Assume μ_n = 1000 cm^2/V-s and T = 300 K.

Solution

The diffusion current is obtained from:

$$J_n = q\,D_n \frac{dn}{dx} = 1.6\ x\ 10^{-19}\ x\ 25.8\ x\ \frac{10^{-17} - 6\ x\ 10^{16}}{2\ x\ 10^{-4}}$$
$$= 828\ A/m^2$$

where the diffusion constant D_n is obtained from:

$$D_n = \mu_n \times V_t = 1000 \times 0.0258 = 25.8\ \text{cm}^2$$

Example 4.1

An abrupt silicon p-n junction consists of a p-type region containing 2 x 10^{16} cm^{-3} acceptors and an n-type region containing also 10^{16} cm^{-3} acceptors in addition to 10^{17} cm^{-3} donors.

a. Calculate the thermal equilibrium density of electrons and holes in the p-type region as well as both densities in the n-type region.

b. Calculate the built-in potential of the p-n junction.

c. Calculate the built-in potential of the p-n junction at 400 K.

Solution a.

The thermal equilibrium densities are:

In the p-type region:

$$p = N_a = 2\ x\ 10^{16} cm^{-3}\ , n = \frac{n_i^2}{p} = \frac{10^{20}}{2\ x\ 10^{16}} = 5\ x\ 10^3 cm^{-3}$$

In the n-type region

$$n = N_d - N_a = 9\ x\ 10^{16} cm^{-3}\ , p = \frac{n_i^2}{n} = \frac{10^{20}}{1\ x\ 10^{16}}$$
$$= 1.11\ x\ 10^3 cm^{-3}$$

b. The built-in potential is obtained from

$$f_i = V_t\, ln \frac{p_p n_n}{n_i^2} = 0.0259\ ln \frac{2\ x\ 10^{16}\ x\ 9\ x\ 10^{16}}{10^{20}} = 0.79V$$

c. Similarly, the built-in potential at 400 K equals

$$f_i = V_t \ln \frac{p_p n_n}{n_i^2} = 0.0345 \ln \frac{2\,x\,10^{16}\,x\,9\,x\,10^{16}}{(4.52\,x10^{12})^2} = 0.63V$$

where the intrinsic carrier density at 400 K was obtained from Example 2.4 b.

Example 1.4

Calculate the diffusion current density for a given semiconductor. Consider silicon at T = 300 K. Assume the electron concentration varies linearly from n = 10^{12} cm^{-3} to n = 10^{16} cm^{-3} over the distance from x = 0 to x = 3 μ$_m$. Assume D$_n$ = 35 cm^2/s.

We have

$$\mathrm{J = e\,D_n \frac{dn}{dx} = e\,D_n \frac{\Delta\,n}{\Delta\,x} = 1\,.6\,x\,10^{-19}\,x\,35\,x \left(\frac{10^{12} - 10^{16}}{0 - 3\,x\,10^{-4}}\right) = 187\ {}^{A}\!/_{m^2}}$$

Comment: Diffusion current densities on the order of a few hundred amperes per square centimeter can also be generated in a semiconductor.

Exercise 1.4

Consider silicon at T = 300 K. Assume the hole concentration is given by p = $10^{16}e^{-x}$/Lp (cm^{-3}), where L$_p$ = 10^{-3} cm. Calculate the hole diffusion current density at (a) x = 0 and (b) x = 10^{-3}cm. Assume D$_p$ = 10 cm^2/s.

Solution: (a) 16 A/cm2, (b) 5.89 A/cm2)

Example 1.7

Determine the current in a pn junction diode. Consider a pn junction at T = 300 K in which I$_S$ = 10^{-14} A and n = 1. Find the diode current for V$_D$ = +0.70 V and V$_D$ = −0.70 V.

Solution:

For VD = +0.70 V, the pn junction is forward-biased and we find

$$\mathrm{i_D = I_s \left[e^{V_D/V_T} - 1\right] = 10^{-14} \left[e^{0.7/0,026} - 1\right] \longrightarrow 4.93\ mA}$$

For $\upsilon_D = -0.7$ V , the pn junction is reverse bias

$$i_D = I_s\left[e^{V_D/V_T} - 1\right] = 10^{-14}\left[e^{-0.7/0{,}026} - 1\right] \cong -10^{-14}\ \mathrm{A}$$

Comment: Although I_S is quite small, even a relatively small value of forward-bias voltage can induce a moderate junction current. With a reverse-bias voltage applied, the junction current is virtually zero.

Exercise 1.7

(a) A silicon pn junction at T = 300 K has a reverse-saturation current of $I_S = 2 \times 10^{-14}$ A. Determine the required forward-bias voltage to produce a current of (i) $I_D = 50$ μA and (ii) $I_D = 1$ mA. (b) Repeat part (a) for $I_S = 2 \times 10^{-12}$

Solution

a. **(i) 0.563 V, (ii) 0.641 V; (b) (i) 0.443 V, (ii) 0.521 V).**

Part (12) Solved PROBLEMS

continuity equation

Problem (1)

Consider n-type silicon with $N_d = 10^{15}\mathrm{cm}^{-3}$ at T = 300◦K. A light source is turned on at t = 0. The source illuminates the semiconductor uniformly, generating carriers at the rate of $G_n = G_p = 10^{19}\mathrm{cm}^{-3}\mathrm{s}^{-1}$. There is no applied field.

(a) Write down the continuity equation and solve it to find the expression for the excess minority carrier concentration, $\delta p(t)$, as a function of time for t ≥ 0.

Solution: When there is no applied electric field the carrier distribution is diffusion driven. The continuity equation for the minority carrier

$$\frac{\partial p(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_p(x,t)}{\partial x} + G_p(x,t) - R_p(x,t)$$

Then reduces to ,

$$\frac{\partial \delta_p}{\partial t} = G_p - \frac{\partial \delta_p}{\tau_p}$$

With the general solution

$$\delta_p(t) = A\, exp\left(-\frac{t}{\pi_p}\right) + G_p\tau_p$$

Using the initial condition (before the light was turned on) that $\delta p(t)$ = 0, then we find that$A = -G_p\tau$. The full solution then is,

$$\delta_p(t) = G_p\tau_p\left\{1 - exp\left(-\frac{t}{\tau_p}\right)\right\}$$

(b) As t →∞, the system will approach steady state. When the steady state excess carrier concentration is 5×10^{13}cm^{-3}, find the minority carrier lifetime, τ_p.

Solution: The system will approach steady state as t → ∞. Evidently, the steady state carrier density is given $\delta p(t)|t \to \infty = G_p\tau_p$. The τ_p must then take on the value $\tau_p = {5\,x\,10^{13}}/{G_p}$. With G$_p$ = 10^{19} cm^{-3}s^{-1}, then the minority (hole) carried lifetime must be τ_p = 5×10^{-6}s.

(c) Determine the time at which the excess carrier concentration becomes half of the steady state value,$\delta p(t)|t \to \infty$ that you calculated in (b).

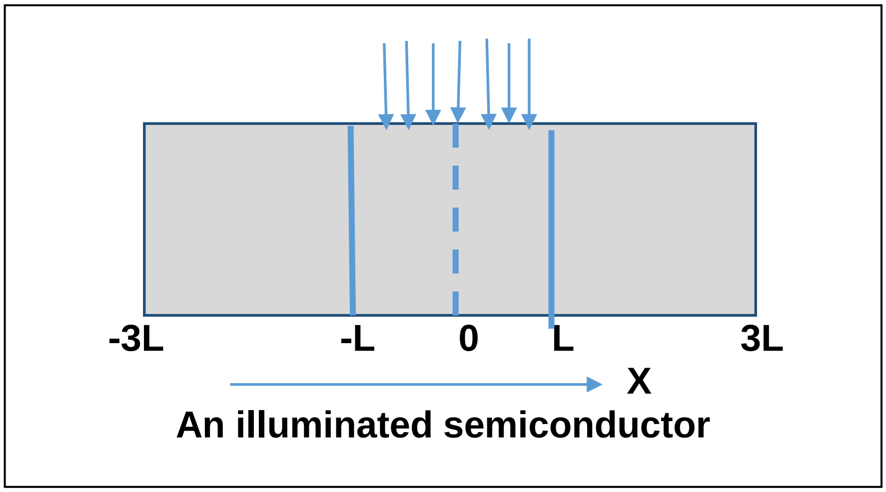
Solution: The value at which

$$exp\left(-\frac{t}{\tau}\right) = \frac{1}{2}\ \ so, t = ln\,(2)\tau_p = 0.69\,x\,5\,x\,10^{-6}\,s$$

Problem (2)

2. Consider an n-type semiconductor as shown. Illumination

produces a constant excess carrier generation rate of G$_p$ is the region −L ≤ x ≤ L. Assume the minority current density is zero at x = −3L and x = 3L. Find the steady state minority carrier concentration as a function of x, $\delta p(x)$. There is no applied electric field.



An illuminated semiconductor

Solution: When there is no applied electric field the carrier distribution is diffusion driven. The continuity equation for the minority carrier

$$\frac{\partial p(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_p(x,t)}{\partial x} + G_p(x,t) - R_p(x,t)$$

Then reduces to,

$$0 = D_p\frac{\partial^2 \delta_p(x,t)}{\partial^2} + G_p - \frac{\delta_p}{\tau_p}$$

In $-L \leq x \leq\leq L$

And

$$0 = D_p\frac{\partial^2 \delta_p(x,t)}{\partial^2} - \frac{\delta_p}{\tau_p}$$

In $-3L \leq x \leq\leq -L$ and $L \leq x \leq\leq 3L$ **, with boundary conditions that**

$$\frac{\partial\, \delta_p}{\partial x}(-3L) = \frac{\partial\, \delta_p}{\partial x}(3\,L) = 0$$

and $\delta p(-L)$ and $\delta p(L)$ are continuous. From symmetry, we need only solve for x > 0 with new boundary condition that

$$\frac{\partial\, \delta_p}{\partial x}(0) = 0$$

The solution is

$$\delta_p(x) = \begin{Bmatrix} G_p\tau_p + A\, exp\left(-\frac{x}{L_p}\right) + B\, exp\left(\frac{x}{L_p}\right) & 0 \le x \le L \\ C\, exp\left(-\frac{x}{L_p}\right) + D\, exp\left(\frac{x}{L_p}\right) & L \le x \le 3L \end{Bmatrix}$$

Where,

$$L_p^2 = D_p\tau_p$$

The end conditions at 0 and L then lead to A = B

And $C = D\, exp\left(\frac{6\,L}{L_p}\right)$

These results in

$$\delta_p(x) = \begin{Bmatrix} G_p\tau_p + E\, cosh\left(\frac{x}{L_p}\right) & 0 \le x \le L \\ F cosh\left(\frac{x - 3L}{L_p}\right) & L \le x \le 3L \end{Bmatrix}$$

Where

$$\mathrm{E} = 2\mathrm{A} \text{ and } \mathrm{F} = 2\,\mathrm{D}\exp\left(\frac{3\,L}{L_p}\right)$$

Are the new constants to find and where

$$2\, cosh\,(x) = exp(x) + exp\,(-x)$$

is the hyperbolic cosine function? We will later also use the derivative of this function.

$$2\, sinh\,(x) = exp(x) + exp\,(-x)$$

The hyperbolic sine function. At x = L, we find

$$G_p\tau_p + E\, cosh\left(\frac{L}{L_p}\right) = F\, cosh\left(\frac{2L}{L_p}\right)$$

$$E\, sinh\left(\frac{x}{L_p}\right) = -\, F\, sinh\left(\frac{2L}{L_p}\right)$$

where we have used the evenness of cosh and oddness of sinh. Solving, we find

$$F = -\frac{sinh\left(\frac{L}{L_p}\right)}{sinh\left(\frac{2L}{L_p}\right)}\ E$$

$$E = -\frac{G_p\tau_p\left(\frac{2L}{L_p}\right)}{cosh\left(\frac{L}{L_p}\right)\ sinh\left(\frac{2L}{L_p}\right) + \ cosh\left(\frac{2L}{L_p}\right)\ sinh\left(\frac{L}{L_p}\right)}$$

$$= \frac{G_p\tau_p sinh\left(\frac{2L}{L_p}\right)}{sinh\left(\frac{3L}{L_p}\right)}$$

Where the identity,

$$sinh\left(\frac{3L}{L_p}\right) = cosh\left(\frac{L}{L_p}\right)\ sinh\left(\frac{2L}{L_p}\right) + \ cosh\left(\frac{2L}{L_p}\right)\ sinh\left(\frac{L}{L_p}\right)$$

has been used. Substituting back, we find

$$F = \frac{G_p\tau_p sinh\left(\frac{L}{L_p}\right)}{sinh\left(\frac{3L}{L_p}\right)}$$

$$\delta_p(x) = \left(1 - \frac{sinh\left(\frac{2L}{L_p}\right)\ cosh\left(\frac{x}{L_p}\right)}{sinh\left(\frac{3L}{L_p}\right)}\right)\ \ 0\ \leq x\ \leq L$$

$$\frac{G_p\tau_p sinh\left(\frac{L}{L_p}\right)\ cosh\left(\frac{x-3L}{L_p}\right)}{sinh\left(\frac{3L}{L_p}\right)}\ \ L\ \leq x\ \leq 3L$$

(b) Suppose the excess carriers are generated such that the

excess carrier concentrations are 10% of the equilibrium majority carrier concentrations. Calculate the quasi-Fermi levels relative the intrinsic Fermi levels, that is, $F_n - E_i$ and $E_i - F_p$.

Solution: Here we can use that

$n_o + \delta_n = n_i \, exp\left(\frac{E_n - E_i}{KT}\right)$ **, to find,**

$$F_n - E_i = KT \, ln\left(\frac{1.1 \, x \, N_d}{n_i}\right) = 0.521 \, eV$$

The minority carriers than are given by

$p_o = \frac{n_i^2}{n_o} = 4 \, x \, 10^{-3} cm^{-3}$ **, And ,**

$$p = p + \delta_p \approx \delta_p = n_i \, exp\left(\frac{E_i - E_F}{KT}\right)$$

That yields,

$$E_i - E_p = KT \, ln\left(\frac{0.1 \, x \, N_d}{n_i}\right) = 0.459 \, eV$$

Part (13) Solved PROBLEMS

PN junction theory

Example

A silicon diode at temperature T = 300K has doping concentration $N_A = N_D = 1.2x10^{16}$ cm $^{-3}$, $n_i = 1.5 \, x10^{10}$ cm $^{-3}$, D_n = 25 cm^2 s $^{-1}$, D_p = 10 cm^2 s $^{-1}$, K_S = 11.7, $L_p = 2.2x10^{-3}$cm and $L_n = 3.5x10^{-3}$cm, and cross section area = $1x10^{-2}$ cm^2 .

Solution:

The minority hole in n - region

$$pno = \frac{n_i^2}{N_A} = \frac{2.25x10^{20}}{1.2x10^{16}} = 1.875x10^4 \, cm^{-3}$$

The minority electron in n_{po} p-region

$$\mathrm{npo} = \frac{\mathrm{n_i^2}}{\mathrm{N_D}} = \frac{2.25\mathrm{x}10^{20}}{1.2\mathrm{x}10^{16}} = 1.875\mathrm{x}10^{4}\ \mathrm{cm}^{-3}$$

The reverse saturation current

$$\mathrm{I_s} = A\left[\frac{q\,D_p p_{no}}{L_p} + \frac{q\,D_n n_{po}}{L_n}\right] = 1.602\ x\ 10^{-19}\left[\frac{10\ x\ 1.0875\ x\ 10^{4}}{2.2\ x\ 10^{-3}} + \frac{25\ x\ 1.875\ x\ 10^{4}}{3.5\ x\ 10^{-3}}\right] = 3.51\ x\ 10^{-13}\ A$$

Example

An abrupt silicon ($n_I = 10^{10}$ cm^{-3}) p-n junction consists of a p-type region containing 10^{16} cm^{-3}acceptors and an n-type region containing 5 x 10^{16} cm^{-3} donors.

a. Calculate the built-in potential of this p-n junction.
b. Calculate the total width of the depletion region if the applied voltage V_a equals 0, 0.5 and -2.5 V.
c. Calculate maximum electric field in the depletion region at 0, 0.5 and -2.5 V.
d. Calculate the potential across the depletion region in the n-type semiconductor at 0, 0.5 and -2.5 V.

Solution:

The built-in potential is calculated from:

$$\phi_i = V_t \ln\frac{p_n n_p}{n_i^2} = 0.0259 \ln\frac{10^{16} \times 5 \times 10^{16}}{10^{20}} = 0.76\ \mathrm{V}$$

The depletion layer width is obtained from:

$$x_d = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(\phi_i - V_a)}$$

the electric field from

$$\mathcal{E}(x=0) = -\frac{2(\phi_i - V_a)}{x_d}$$

and the potential across the n-type region equals

$$\phi_n = \frac{qN_d x_n^2}{2\varepsilon_s}$$

Where

$$x_n = x_d \frac{N_a}{N_a + N_d}$$

One can also show that:

$$\phi_n = \frac{(\phi_i - V_a)N_a}{N_a + N_d}$$

This yields the following numeric values:

	$V_a = 0$ V	$V_a = 0.5$ V	$V_a = -2.5$ V
x_d	0.315 μm	0.143 μm	0.703 μm
$\mathcal{E}$	40 kV/cm	18 kV/cm	89 kV/cm
ϕ_n	0.105 V	0.0216 V	0.522 V

Example:

Consider an abrupt p-n diode with $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. Calculate the junction capacitance at zero bias. The diode area equals 10^{-4} cm^2. Repeat the problem while treating the diode as a one-sided diode and calculate the relative error.

Solution:

The built in potential of the diode equals:

$$\phi_i = V_t \ln \frac{N_d N_a}{n_i^2} = 0.83 \text{ V}$$

The depletion layer width at zero bias equals:

$$x_d = \sqrt{\frac{2\varepsilon_s(\phi_i - 0)}{qN_d}} = 0.33 \ \mu\text{m}$$

And the junction capacitance at zero bias equals:

$$C_{j0} = \frac{\varepsilon_s}{x_d}\bigg|_{V_a=0} = 3.17\,\text{pF}$$

Repeating the analysis while treating the diode as a one-sided diode, one only has to consider the region with the lower doping density so that

$$x_d \cong x_n = \sqrt{\frac{2\,\varepsilon_s}{qN_d}(\phi_i - V_a)} = 0.31\,\mu\text{m}$$

And the junction capacitance at zero bias equals

$$C_{j0} = \frac{\varepsilon_s}{x_d}\bigg|_{V_a=0} = 3.18\,\text{pF}$$

The relative error equals 0.5 %, which justifies the use of the one-sided approximation.

Example

An abrupt silicon p-n junction ($N_a = 10^{16}$ cm^{-3} and $N_d = 4 \times 10^{16}$ cm^{-3}) is biased with $V_a = 0.6$ V. Calculate the ideal diode current assuming that the n-type region is much smaller than the diffusion length with $w_n' = 1$ mm and assuming a "long" p-type region. Use $m_n = 1000$ cm^2/V-s and $m_p = 300$ cm^2/V-s. The minority carrier lifetime is 10 ms and the diode area is 100 mm by 100 mm.

Solution:

The current is calculated from:

$$I = qA\left[\frac{D_n n_{p0}}{L_n} + \frac{D_p p_{n0}}{w_n'}\right]\left(e^{V_a/V_t} - 1\right)$$

With

$D_n = m_n V_t = 1000 \times 0.0258 = 25.8$ cm^2/V-s

$D_p = m_p V_t = 300 \times 0.0258 = 7.75$ cm^2/V-s

$n_{p0} = n_i^2/N_a = 10^{20}/10^{16} = 10^4$ cm^{-3}

$p_{n0} = n_i^2/Nd = 10^{20}/4 \times 10^{16} = 2.5 \times 10^3$ cm^{-3}

$$L_n = \sqrt{D_n \tau_n} = \sqrt{25.8 \times 10^{-5}} = 161\,\mu\text{m}$$

Yielding I = 40.7 mA

Note that the hole diffusion current occurs in the "short" *n*-type region and therefore depends on the quasi-neutral width in that region. The electron diffusion current occurs in the "long" *p*-type region and therefore depends on the electron diffusion length in that region.

Example

b. Calculate the diffusion capacitance of the diode described in Example 4.4 at zero bias. Use $\mu_n = 1000$ cm^2/V-s, $\mu_p = 300$ cm^2/V-s, $\boldsymbol{w_p'} = 1$ µm and $w_n' = 1$ mm. The minority carrier lifetime equals 0.1 ms.

c. For the same diode, find the voltage for which the junction capacitance equals the diffusion capacitance.

Solution

a. The diffusion capacitance at zero volts equals

$$C_{d,0} = \frac{I_{s,p}\tau_p}{V_t} + \frac{I_{s,n}t_{r,n}}{V_t} = 1.73\times10^{-19}\ \mathrm{F}$$

Using

$$I_{s,p} = q\frac{A p_{n0} D_p}{L_p}$$

And

$$I_{s,n} = q\frac{A n_{p0} D_n}{w_p'}$$

Where the "short" diode expression was used for the capacitance associated with the excess charge due to electrons in the p-type region. The "long" diode expression was used for the capacitance associated with the excess charge due to holes in the n-type region.The diffusion constants and diffusion lengths equal

$$D_n = \mu_n \times V_t = 25.8\ \mathrm{cm^2/s}$$

$$D_p = \mu_p \times V_t = 7.75\ \mathrm{cm^2/s}$$

$$L_p = \sqrt{D_p\tau_p}$$

And the electron transit time in the p-type region equals

$$t_{r,n} = \frac{w_p'^2}{2D_n} = 193\,\text{ps}$$

b. The voltage at which the junction capacitance equals the diffusion capacitance is obtained by solving

$$\frac{C_{j0}}{\sqrt{1-\frac{V_a}{\phi_i}}} = C_{d,0}e^{V_a/V_t}$$

Yielding $V_a = 0.442$ V

Part (14) Solved PROBLEMS ideal and Nonideal p-n Junction

1. Find the built-in potential for a p-n Si junction at room temperature if the bulk resistivity of Si is 1 Ωcm. Electron mobility in Si at RT is 1400 cm^2 V^{-1} s^{-1}; $\mu_n/\mu_p = 3.1$; $n_i = 1.05\times10^{10}$ cm^{-3}.

Solution:

By definition, eφ_d = Fn −Fp. Concentrations of the free carriers are given by

$$n = N_c\left(-\frac{E_g - F_n}{KT}\right) \quad , \qquad p = N_v\, exp\left(-\frac{F_p}{KT}\right)$$

From here we get that

$$e\,\varphi_d = E_g + KT\,ln\left(\frac{n}{N_c}\right) + KT\,ln\left(\frac{p}{N_v}\right) = E_g + KT\,ln\left(\frac{n\,p}{N_c N_v}\right)$$

$$n_i^2 = N_c N_v\, exp\left(-\frac{E_g}{KT}\right)$$

We obtain ,

$$\varphi_d = \frac{KT}{e}\,ln\left(\frac{n\,p}{n_i^2}\right)$$

From n = 1/eρμ$_n$ and p = 1/eρμ$_p$, we finally get φ_d = 0.68 V.

2. For the p-n Si junction from the previous problem calculate the width of the space charge region for the applied voltages V = −10, 0, and +0.3 V. $\epsilon_{Si} = 11.9$

Solution:

Taking into account that at room temperature all donors and acceptors are ionized, i.e. $n = N_d$ and $p = N_a$, from the values found in the previous problem and

$$\boldsymbol{\omega = \left\{\frac{\epsilon\,(\varphi_d - V)}{2\,\pi\,e}\frac{N_d + N_A}{N_d N_A}\right\}^{1/2}}$$

we get ω(−10V) = 2 μm, ω(0V) = 0.5 μm, and ω(+0.3V) = 0.4 μm.

1. For the parameters given in the previous problem find the maximum electric field within the space charge region. Compare these values with the electric field within a shallow donor: $E \approx e/\,\epsilon_{Si}\, a^2_B$, where a_B is the Bohr radius of a shallow donor,

$$\boldsymbol{a_b = \frac{\epsilon_{si} h^2}{m_e^* e^2}\ and\ \frac{m_e^*}{m_o} = 0.33}$$

Solution:

From the previous problem and

$$\boldsymbol{E = 2\left\{\frac{2\,\pi\,e\,(\varphi_d - V)}{\epsilon}\frac{N_d + N_A}{N_d N_A}\right\}^{1/2}}$$

we obtain that E(−10V) = 10^5 V/cm, E(0V) = 2.6×10^4 V/cm, and E(+0.3V) = 2×10^4 V/cm.

The electric field within a shallow donor is, in turn, E ≈ 3.4×10^5 V/cm, that is, comparable to that of the p-n junction.

4. Calculate the capacity of the p-n junction from the problem 2 if the area of the junction is 0.1 cm^2.

Solution:

Since

$$\boldsymbol{C = \frac{\epsilon\,S}{4\,\pi\,\omega}}$$

we get C(−10V) = 0.5 nF, C(0V) = 2 nF, and C(+0.3V) = 2.6 nF.

5. n-Si of a p-n Si junction has a resistivity of 1 Ωcm. What should be the resistivity of p-Si so that 99 % of the total width of the space charge region would be located in n-Si (p^+-n junction)? For the parameters needed see problem 1.

Solution:

From the conditions of the problem $\omega_a = 0.01\omega$ and $\omega_d = 0.99\omega$. Since $\omega_a/\omega_d = N_d/N_a$, we get that Na = $99N_d$. Because $N_d = 1/e\rho\mu_n = 4.5\times10^{15}$ cm^{-3}, we get Na = 4.4×10^{17} cm^{-3}.

6. At room temperature under the forward bias of 0.15 V the current through a p-n junction is 1.66 mA. What will be the current through the junction under reverse bias?

Solution:

j_s = 1.66mAexp(−eV/kT) = 4 μA.

7. For a p^+-n Si junction the reverse current at room temperature is 0.9 nA/cm^2. Calculate the minority-carrier lifetime if $N_d = 10^{15}$ cm^{-3}, $n_i = 1.05\times10^{10}$ cm^{-3}, and $\mu_p = 450$ cm^2 V^{-1} s^{-1}.

Solution:

For a p^+-n junction

$$J_s = \frac{e\,p\,D_p}{L_p} = \frac{e\,D_p n_i^2}{N_d L_p} = \frac{e\,n_i^2}{N_d}\left(\frac{D_p}{\tau_p}\right)^{1/2}$$

Taking into account that μ = eD/kT, we finally get $\tau_p = 4.5\times10^{-9}$ s.

8. How does the reverse current of a Si p-n junction change if the temperature raises from 20 to 50 ◦C? The same for a Ge p-n junction. Band gaps of Si and Ge are 1.12 and 0.66 eV, respectively.

Solution:

Since$j_s \sim n_i^2 \sim T^3\, exp\left(-\frac{E_g}{KT}\right)$

we get

$$\frac{j_s(T_2)}{j_s(T_1)} = \left(\frac{T_2}{T_1}\right)^3\ exp\left(-\frac{E_g}{KT_2} + \frac{E_g}{KT_1}\right)$$

From here the ratios of the reverse currents in the p-n junctions made

of Ge and Si are 15 and 82, respectively.

9. Estimate temperatures at which p-n junctions made of Ge, Si, and GaN lose their rectifying characteristics. In all cases $N_a = N_d = 10^{15}$ cm^{-3}. Assume that E_g are independent of the temperature and are 0.66, 1.12, and 3.44 eV for Ge, Si, and GaN, respectively. Intrinsic carrier concentrations at room temperature are $n_i^{Ge} = 2\times10^{13}$, $n_i^{Si} = 10^{10}$, and $n_i^{GaN} = 10^{-9}$ cm^{-3}.

Solution:

p-n junction stops working when concentrations of electrons and holes equalize. It happens when

$$N_d(N_A) \cong n_i \approx \sqrt{N_c N_v}\, exp\left(\frac{-E_g}{2\,KT}\right) \sim T^{3/2}\, exp\left(\frac{-E_g}{2\,KT}\right)$$

From here and the parameters given we get that the maximum temperatures are $T_{Ge} \approx 400$ K, $T_{Si} \approx 650$ K, and $T_{GaN} \approx 1700$ K. That is, only wide band gap semiconductors are suitable for extremal applications.

Problem 2.25

Electrons in silicon carbide have a mobility of 1000 cm^2/V-sec. At what value of the electric field do the electrons reach a velocity of 3 x 10^7 cm/s? Assume that the mobility is constant and independent of the electric field. What voltage is required to obtain this field in a 5 micron thick region? How much time do the electrons need to cross the 5 micron thick region?

Solution The electric field is obtained from the mobility and the velocity:

$$\varepsilon = \frac{\mu}{\nu} = \frac{1400}{3\,x\,10^7} = 30\,KV/cm$$

Combined with the length one finds the applied voltage.

$$V = \varepsilon\, L = 30{,}000 \text{ x } 5 \text{ x } 10^{-4} = 15 \text{ V}$$

The transit time equals the length divided by the velocity:

$$t_r = L/\nu = 5 \text{ x } 10^{-4}/3 \text{ x } 10^7 = 16.7 \text{ ps}$$

Example 4.2

An abrupt silicon ($n_i = 10^{10}$ cm^{-3}) p-n junction consists of a ptype region containing 10^{16} cm^{-3} acceptors and an n-type region containing 5 x 10^{16} cm^{-3} donors.

a. Calculate the built-in potential of this p-n junction.

b. Calculate the total width of the depletion region if the applied voltage V_a equals 0, 0.5 and -2.5 V.

c. Calculate maximum electric field in the depletion region at 0, 0.5 and -2.5 V.

d. Calculate the potential across the depletion region in the n-type semiconductor at 0, 0.5 and -2.5 V.

Solution

The built-in potential is calculated from:

$$f_i = V_t \, ln \, \frac{p_n n_p}{n_i^2} = 0.0259 \, ln \, \frac{10^{16} \, x \, 5 \, x \, 10^{16}}{10^{20}} = 0.76V$$

The depletion layer width is obtained from:

$$w = \sqrt{\frac{2 \, e_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(f_i - V_a)}$$

the electric field from

$$\varepsilon \, (x = 0) = \frac{2 \, (f_i - V_a)}{w}$$

and the potential across the n-type region equals

$$f_n = \frac{q \, N_d x_n^2}{2 \, e_s}$$

Where,

$$x_n = w \, \frac{N_a}{N_a + N_d}$$

one can also show that

$$f_n = \frac{(f_i - V_a) N_a}{N_a + N_d}$$

This yields the following numeric values:

	V_a = 0 V	V_a = 0.5 V	V_a = -2.5 V
w	0.315 µm	0.143 µm	0.703 µm
ε	40 kV/cm	18 kV/cm	89 kV/cm
f_n	0.105 V	0.0216	V 0.522 V

Example 4.3

An abrupt silicon p-n junction ($N_a = 10^{16}$ cm^{-3} and $N_d = 4 \times 10^{16}$ cm^{-3}) is biased with V_a = 0.6 V. Calculate the ideal diode current assuming that the n-type region is much smaller than the diffusion length with w_n = 1 µm and assuming a "long" p-type region. Use μ_n = 1000 cm^2/V-s and μ_p = 300 cm^2/V-s. The minority carrier lifetime is 10 µs and the diode area is 100 µm by 100 µm.

Solution:

The current is calculated from:

$$I = q\,A\left[\frac{D_n n_{po}}{L_n} + \frac{D_p p_{no}}{W_n}\right]\left(e^{V_o/V_t} - 1\right)$$

with

$D_n = \mu_n V_t$ = 1000 x 0.0258 = 25.8 cm^2/V-s

$D_P = \mu_P V_t$ = 300 x 0.0258 = 7.75 cm^2/V-s

$n_{p0} = n^2_i/N_a = 10^{20}/10^{16} = 10^4$ cm^{-3}

$p_{n0} = n^2_i/N_d = 10^{20}/4 \times 10^{16} = 2.5 \times 10^3$ cm^{-3}

$$L_n = \sqrt{D_n \tau_n} = \sqrt{25.8\, x\, 10^{-5}} = 161\ \mu m$$

yielding I = 40.7 µA

Note that the hole diffusion current occurs in the "short" n-type region and therefore depends on the quasi-neutral width in that region. The electron diffusion current occurs in the "long" p-type region and therefore depends on the electron diffusion length in that region.

Example 4.4
Consider an abrupt p-n diode with $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. Calculate the junction capacitance at zero bias. The diode area equals 10^{-4} cm^2. Repeat the problem while treating the diode as a one-sided diode and calculate the relative error.

Solution The built in potential of the diode equals:

$$f_i = V_t \, ln \, \frac{N_d N_a}{n_i^2} = 0.83V$$

The depletion layer width at zero bias equals:

$$W = \sqrt{\frac{2 \, e_s (f_i - 0)}{q \, N_d}} = 0.33 \, \mu m$$

And the junction capacitance at zero bias equals:

$$C_{jo} = \left. \frac{e_s}{W} \right|_{va=0} = 3.17 \, pF$$

Repeating the analysis while treating the diode as a one-sided diode, one only has to consider the region with the lower doping density so that

$$W = x_n = \sqrt{\frac{2 \, e_s}{q \, N_d} (f_i - V_a)} = 0.31 \, \mu m$$

And the junction capacitance at zero bias equals

$$C_{jo} = \left. \frac{e_s}{W} \right|_{va=0} = 3.18 \, pF$$

The relative error equals 0.5 %, which justifies the use of the one-sided approximation.

1.3 Ideal p-n Junction

1. Find the built-in potential for a *p-n* Si junction at room temperature if the bulk resistivity of Si is 1 Ώcm. Electron mobility in Si at RT is 1400 cm^2 V^{-1} s^{-1}; $\mu_n/\mu_p = 3.1$; $n_i = 1.05 \times$

10^{10} cm^{-3}.

Solution: (3.1)
By definition, eφ_d = F_n − F_p. Concentrations of the free carriers are given by

$$n = N_c\, exp\left(-\frac{E_g - F_n}{KT}\right), N_v\, exp\left(-\frac{E_g - F_p}{KT}\right)$$

From here we get that

$$e\varphi_d = E_g + KT\, ln\left(\frac{n}{N_c}\right) + KT\, ln\left(\frac{p}{N_v}\right) = E_g + KT\, ln\left(\frac{n\,p}{N_c N_v}\right)$$

Since,

$$n_i^2 = N_c\, N_v\, exp\left(-\frac{E_g}{KT}\right)$$

we obtain that

$$\varphi_d = \frac{KT}{e}\, ln\left(\frac{np}{n_i^2}\right)$$

From n = 1/e ρ μ_n and p = 1/e ρ μ_p, we finally get φ_d = 0.68 V.

6. At room temperature under the forward bias of 0.15 V the current through a *p-n* junction is 1.66 mA. What will be the current through the junction under reverse bias?

Solution: (3.6)
j_s = 1.66 mA exp(−eV/kT) = 4 μA.

7. For a *p_+-n* Si junction the reverse current at room temperature is 0.9 nA/cm^2. Calculate the minority-carrier lifetime if N_d = 10^{15} cm^{-3}, n_i = 1.05 × 10^{10} cm^{-3}, and μ_p = 450 cm^2 V^{-1} s^{-1}.

Solution: (3.7)
For a *p_+-n* junction

$$J_S = \frac{e\, p\, D_p}{L_p} = \frac{e\, n_i^2\, D_p}{N_d L_p} = \frac{e\, n_i^2}{N_d}\left(\frac{D_p}{\tau_p}\right)^{1/2}$$

Taking into account that μ = eD/kT, we finally get $\tau_p = 4.5 \times 10^{-9}$ s.

1. Problem: A silicon p-n junction is formed between n-type Si doped with $N_D = 10^{17}$ cm^{-3}and p-type Si doped with NA = 10^{16} cm^{-3}.

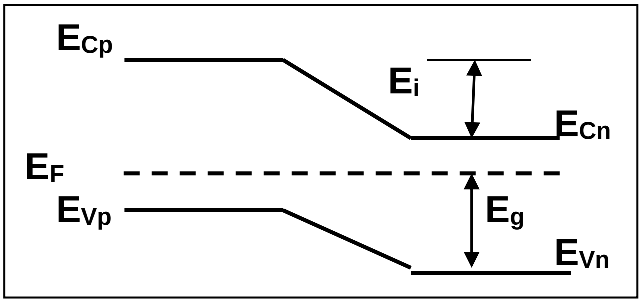
(a) Sketch the energy band diagram. Label all axes and all important energy levels.

(b) Find n_{n0}, n_{p0}, p_{p0}, and $_{pn0}$. Sketch the carrier concentration (of both electrons and holes) as a function of position.

(c) Calculate the built-in potential V_{bi} in eV.

Solution:

(a) The energy band diagram with labeled important energy levels and axes is



(b) Given *n*+= 1.5 x 10^{10} cm^{-3},

in the quasi-neutral p-region,

$$P_{po} = N_A = 10^{16}\ \mathrm{cm}^{-3}$$

In the quasi-neutral n-region,

$$\mathbf{n_{no} = N_D = 10^{17}\ cm^{-3}}$$

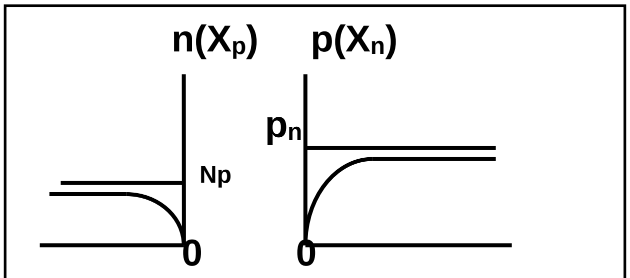
In the depletion region,

$$\mathbf{n_{po} = \frac{n_i^2}{N_A} = 2.25\ x10^4\ cm^{-3}}$$

for p-side and

$$p_{no} = \frac{n_i^2}{N_D} = 2.25\ x10^3\ cm^{-3}$$

for n-side.. The diagram for carrier concentration is:



(b) The build-in potential is

$$V_{bi} = \frac{KT}{q}\ln\frac{N_A N_D}{n_i^2} = \frac{KT}{q}\ln\frac{p_p}{p_n} = 0.754\ \text{eV}$$

Problem: 4

Consider a p$^+$-n Si junction at *T* = 300 K with $N_A = 10^{18}$ cm^{-3} and $N_D = 10^{16}$ cm^{-3}. The minority carrier hole diffusion constant is $D_p = 12$ cm^2/s and the minority carrier hole lifetime is $\tau_p = 100$ ns. The cross-sectional area of the junction is $A = 10^{-4}$ cm^2. Calculate the reverse saturation current *Is* = A_{Js}. Calculate also the current at a forward bias $V_a = 0.5$ V.

Solution:

Since $N_A >> N_D$, it is an asymmetric junction and the total current is dominated by the most heavily-doped side of the junction. the saturation current density is given by:

$$J_s = \frac{q\,D_p\,p_{no}}{L_p}$$

Where,

$$L_p = \left(D_p\tau_p\right)^{1/2} = 1.095 \times 10^{-3}\text{cm}^{-3} \text{ and } p_{no} = \frac{n_i^2}{N_D} = 2.25 \times 10^{-3}\text{cm}^{-3}$$

So, $\quad$ Js = 3:9492 x 10^{-11} A/cm^2

Is = - A J_s = - 3:9492 x 10^{-15} A

For a forward bias $V_a = 0.5$ V, the current is:

$$I = I_s\left(e^{qV_o/KT} - 1\right) = 8.93\ \mu A$$

Problem (1)

1. n-Si with $N_d = 7 \times 10^{15}$ cm^{-3} additionally contains $N_t = 10^{15}$ cm^{-3} generation- recombination centers located at the intrinsic Fermi level

with $\sigma_n = \sigma_p = 10^{-15}$ cm^2 and $v_t = 10^7$ cm/s. Calculate generation rate, if
1. n and p are low as compared to the equilibrium value
2. only p is below the equilibrium value. For Si, $n_i = 1.05\times10^{1010}$ cm^{-3}.

Solution :
By definition

$$G_n = -R_n = \frac{n_i^2 - p\,n}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

Where $\tau_n^{-1} = \tau_p^{-1} = N_t\sigma_n v_t = 10^7 s^{-1}$. In the first case n and p are less than n_i. Thus, np <n_i^2 and hence

$$G_n = \frac{n_i^2}{\tau_n(n_i + p_i)} = \frac{n_i}{2\,\tau_n} = 5.3\ x\ 10^{16} cm^{-3}\ /s$$

In the second case $n = N_d \gg n_i$, whereas $p < n_i$, hence

$$G_n = \frac{n_i^2}{\tau_n(n + n_i + p_i)} = \frac{n_i^2}{\tau_n\, n} = \frac{n_i^2}{\tau_n N_d} = 1.6\ x\ 10^{11} cm^{-3}\ /s$$

Problem (2)

Illumination of n-type Si ($N_d = 10^{16}$ cm^{-3}) generates 10^{21} cm^{-3}/s electron-hole pairs. Si has $N_t = 10^{15}$ cm^{-3} generation-recombination centers with σn = σp = 10^{-16} cm^2. Calculate equilibrium concentration of electrons and holes if $E_t = E_i$, where E_i is the Fermi level of intrinsic Si, and $v_t = 10^7$ cm/s.

Solution:
In equilibrium, the generation G = 10^{21} cm^{-3}/s and recombination R rates are equal

$$G_n = R = \frac{n\,p - n_i^2}{\tau_p(n+n_i) + \tau_n(p+p_i)} \approx \frac{n\,p}{\tau_p\, n + \tau_n\, p} = \frac{n\,p}{\tau\,(n + p)}$$

Here we used $\tau_n = \tau_p = \tau = (N_t v_t \sigma_n)^{-1} = 10^{-6}$ sec. In n-type Si under illumination, $n = N_d + \Delta n$, $p \approx \Delta p = \Delta n$. Thus,

$$G_T = \frac{(N_d + \Delta n)\,\Delta n}{N_d + 2\,\Delta n}$$

Solving this equation with respect to Δn we obtain p = Δn = 1.1×10^{15} cm^{-3} and n = 1.1×10^{16} cm^{-3}.

SOLVED PROBLEMS

The reverse saturation current at 3ooK of a p-n Ge diode is 5 uA . Find

the voltage to be applied across the junction to obtain a forward current of 50 mA.

Solution

$$I = I_s \left[exp\left(\frac{e\,V}{KT}\right) - 1\right]$$

So,

$$exp\left(\frac{e\,V}{KT}\right) = \frac{I}{I_s} + 1 = \frac{50\ x\ 10^{-3}}{5\ x\ 10^{-6}} + 1 = 10^4$$

Or ,

$$V = \frac{KT}{e}\ ln\ 10^4 = \frac{1.38\ x\ 10^{-23}\ x\ 300}{1.6\ x\ 10^{-19}}\ x\ 2.303\ x\ 4 = 0.238\ Volt$$

Problem (3)

3. A p^+-n Si junction ($n_i = 1.05\times10^{10}$ cm^{-3}, $\epsilon_{Si} = 11.9$) is formed in an n-type substrate with $N_d = 10^{15}$ cm^{-3}. If the junction contains 10^{15} cm^{-3} generation-recombination centers located at the intrinsic Fermi level with $\sigma_n = \sigma_p = 10^{-15}$ cm^2 ($v_t = 10^7$ cm/s), calculate generation current density at a reverse bias of 10 V.

Solution:

Generation current in the space charge region w is given by

$$j_g = \frac{e\,n_i\,\omega}{2\,\tau}$$

Here, $\tau^{-1} = N_t\sigma_n v_t = 10^7$ s^{-1}. The width w of the space charge region for a p^+-n junction under reverse bias is

$$\omega = \left\{\frac{\epsilon\,(\varphi_d - V)}{2\,\pi\,eN_d}\right\}^{1/2} \approx \left\{\frac{\epsilon\,(V)}{2\,\pi\,e\,N_d}\right\}^{1/2} = 3.6\ \mu m$$

Here we used relation $|V| \gg \varphi_d$. From here we obtain that $j_g = 3\ \mu A/cm^2$.

Problem (4)

For a p-n Si junction with the p-side doped to 10^{17} cm^{-3}, the n-side doped to 10^{19} cm−3 (n^+-p junction), and a reverse bias of −2 V, calculate the generation current density at room temperature,

assuming that the effective lifetime is 10^{-5} s.

Solution:

Using the formulae of the previous problem and relation

$$\varphi_d = \frac{KT}{e}\, ln\left(\frac{N^2}{n_i^2}\right)$$

we get j_s = 1.6 nA/cm^2.

Problem (5)

For a p-n GaAs junction at room temperature find the donor/acceptor concentration at which de Broglie wavelength

$$\left(\lambda = \frac{2\,\pi\, h}{\sqrt{2\, m^*\, E}}\right)$$

$$\left(E = \frac{3KT}{2}\right), \frac{m_e^*}{m_o} = 0.63\,, \frac{m_h^*}{m_o} = 0.53\;\; and\; \epsilon_{GaAs} = 12.9\,, n_i^{GaAs} = 2.1\, x\, 10^6 cm^{-3}\;\; and\; N_A = N_D$$

Solution:

From the parameters given, we find λn = 2.5×10^{-6} cm and λp = 8.5×10^{-7} cm. If N_d = N_a = N the width of the space charge region is

$$\omega = \left\{\frac{\epsilon\varphi_d}{\pi\, e\, N}\right\}^{1/2}$$

By definition, w = λ. Substituting

$$\varphi_d = \frac{KT}{e}\, ln\left(\frac{N^2}{n_i^2}\right)$$

into the expression above and after some simplifications, we get

$$N = \frac{\epsilon}{\pi\, e\, \lambda^2}\frac{2\, KT}{e}\, ln\left(\frac{N}{n_i}\right)$$

Solving the above equation numerically, we obtain N = 6.8×10^{18} cm^{-3} and 6.2×10^{19} cm^{-3} for electron and holes, respectively.

Problem (6)

When a silicon p$^+$-n junction is reverse-biased to 30 V, the depletion-layer capacitance is 1.75 nF/cm^2. If the maximum electric field at avalanche breakdown is 3×10^5 V/cm, find the breakdown voltage. ϵ_{Si} = 11.9.

Solution:

Since C = ϵ /4πw$_0$, and under strong reverse bias w$_0$ ≈ (ϵ V/2πeN$_d$)$^{1/2}$, we obtain N$_d$ = 1.1×10^{15} cm^{-3}. Maximum electric field is at the interface and for a p$^+$-n junction equals E ≈ 4πeN$_d$w$_1$/ϵ. From conditions of the problem we find that at the breakdown w$_1$ = 18 μm and, hence, the breakdown voltage is 273 V .

Problem (7)

For a p$^+$-n Si junction with N$_d$ = 10^{16} cm^{-3}, the breakdown voltage is 32 V. Calculate the maximum electric field at the breakdown. ϵ_{Si} = 11.9

Solution:

The width of the space charge region is w ≈ (ϵ V/2πeN$_d$)$^{1/2}$ = 2 μm. From here we get that the maximum electric field at the breakdown is

$$E = \frac{4\,\pi\, e\, N_d}{\epsilon}\,\omega = 3\ x\ 10^5\, {}^{V}\!/_{cm}$$

1.4 Non ideal p-n Junction

1. n-Si with N$_d$ = 7 × 10^{15} cm^{-3} additionally contains N$_t$ = 10^{15} cm^{-3} generation recombination centers located at the intrinsic Fermi level with σ$_n$ = σ$_p$ = 10^{-15} cm^2 and v$_t$ = 10^7 cm/s. Calculate generation rate, if

1. n and p are low as compared to the equilibrium value

2. only p is below the equilibrium value. For Si, n$_i$ = 1.05 × 10^{10} cm^{-3}.

Solution: (4.1)

By definition

$$G_n = -R_n = \frac{n_i^2 - p_n}{\tau_p\,(n + n_i) + \tau_n\,(p + p_i)}$$

where

$\tau_n^{-1} = \tau_p^{-1} = n_t\sigma_n v_t = 10^7/sec$ In the first case n and p are less than n$_i$. Thus, np < n_i^2 and hence ,

$$G_n = \frac{n_i^2}{\tau_n\,(n_i + p_i)} = \frac{n_i}{2\,\tau_n} = 5{,}3\ x\ 10^{16}\, {}^{cm^{-3}}\!/_{sec}$$

In the second case n = N$_d$ ≫ n$_i$, whereas p < n$_i$, hence

$$G_n = \frac{n_i^2}{\tau_n\ (n + \ n_i + \ p_t)} = \frac{n_i^2}{n\,\tau_n} = \frac{n_i^2}{\tau_n N_d} = 1,6\ x\ 10^{11}\, {cm^{-3}}/_{sec}$$

2. Illumination of n-type Si (N_d = 10^{16} cm^{-3}) generates 10^{21} cm^{-3}/s electron-hole pairs. Si has N_t = 10^{15} cm^{-3} generation-recombination centers with σn = σp = 10^{-16} cm2. Calculate equilibrium concentration of electrons and holes if E_t = E_i, where E_i is the Fermi level of intrinsic Si, and v_t = 10^7 cm/s.

Solution: (4.2)

In equilibrium, the generation G = 10^{21} cm^{-3}/s and recombination R rates are equal,

$$G \quad = \quad R \quad = \frac{n\,p - n_i^2}{\tau_p\,(n + \ n_i) + \tau_n\,(p + \ n_i)} \approx \frac{n\,p}{\tau_p n \ + \tau_n p}$$

$$= \ (n\,p)/\tau(\,n \ + p)$$

Here we used $\tau_n = \tau_p = \tau = (N_t v_t \sigma_n)^{-1} = 10^{-6}$ sec.

In n-type Si under illumination, n = N_d + Δn , p ≈ Δp = Δn. Thus,

$$G_T = \frac{(N_d + \Delta\,n)\Delta n}{N_d + \ 2\,\Delta\,n}$$

Solving this equation with respect to Δn we obtain p = Δn = 1.1×10^{15} cm^{-3} and n = 1.1×10^{16} cm^{-3}.

3. A p^+-n Si junction (n_i = 1.05×10^{10} cm^{-3}, ε = 11.9) is formed in an n-type substrate with N_d = 10^{15} cm^{-3}. If the junction contains 10^{15} cm^{-3} generation-recombination centers located at the intrinsic Fermi level with σn = σp = 10^{-15} cm^2 (v_t = 10^7 cm/s), calculate generation current density at a reverse bias of 10 V.

Solution: (4.3)

Generation current in the space charge region w is given by

$$j_g = \frac{e\,n_i\,w}{2\,\tau}$$

Here, $\tau^{-1} = N_t\,\sigma_n\, v_t = 10^7\ s^{-1}$. The width w of the space charge region for a p^+-n junction under reverse bias is

$$w = \left(\frac{\epsilon\,(\varphi_d - V)}{2\,\pi\,N_d}\right)^{1/2} \approx \left(\frac{\epsilon\,|\mathrm{V}\,|}{2\,\pi\,e\,N_d}\right)^{1/2} = 3.6\ \mu m$$

Here we used relation $|\mathrm{V}\,| \gg \varphi_d$. From here we obtain that $j_g = 3\ \mu A/cm^2$.

4. For a *p-n* Si junction with the p-side doped to $10^{17}\ cm^{-3}$, the n-side doped to $10^{19}\ cm^{-3}$ (n^+-p junction), and a reverse bias of −2 V, calculate the generation current density at room temperature, assuming that the effective lifetime is 10^{-5} s.

Solution: (4.4)

Using the formulae of the previous problem and relation

$$\varphi_d = \frac{KT}{e}\,ln\left(\frac{n\,p}{n_i^2}\right)$$

we get $j_s = 1.6\ nA/cm^2$.

5. For a *p-n* GaAs junction at room temperature find the donor/acceptor concentration at which de Broglie wavelength ($\lambda = 2\pi h\ /\sqrt{2m^* E}$) of electrons/holes is equal to the width of the space charge region. Assume <Ei > 3 kT/2, $m^*_e/m_0 = 0.063$, $m^*_h/m_0 = 0.53$, and $\varepsilon_{GaAs} = 12.9$, $n^{GaAs}_i = 2.1 \times 10^6\ cm^{-3}$, and $N_a = N_d$.

Part (14) Solved PROBLEMS

The transition capacitance , The diffusion capacitance

Example 4.5

a. Calculate the diffusion capacitance of the diode described

in Example 4.4 at zero bias. Use μ$_n$ = 1000 cm^2/V-s, μ$_p$ = 300 cm^2/V-s, w$_p$' = 1 μm and w$_n$' = 1 mm. The minority carrier lifetime equals 0.1 ms.

b. For the same diode, find the voltage for which the junction capacitance equals the diffusion capacitance.

Solution

a. The diffusion capacitance at zero volts' equals

$$C_{d,0} = \frac{I_{s,p}\tau_p}{V_t} + \frac{I_{s,n}\tau_n}{V_t} = 1.73\ x\ 10^{-19}F$$

Using

$$I_{s,p} = q\ \frac{Ap_{no}D_p}{L_p}$$

And ,

$$I_{sn} = q\ \frac{An_{po}D_n}{W_p}$$

Where the "short" diode expression was used for the capacitance associated with the excess charge due to electrons in the p-type region. The "long" diode expression was used for the capacitance associated with the excess charge due to holes in the n-type region. The diffusion constants and diffusion lengths equal

D_n = μ$_n$ x V_t = 25.8 cm^2/s

D_p = μp x V_t = 7.75 cm^2/s

$$L_p = \sqrt{D_p\tau_p}$$

And the electron transit time in the p-type region equals

$$\tau_{r,n} = \frac{w_p^2}{2\ D_n} = 193\ ps$$

b. The voltage at which the junction capacitance equals the diffusion capacitance is obtained by solving

$$\frac{C_{jo}}{\sqrt{1-\frac{V_a}{f_i}}} = C_{d,0}e^{V_m/V_t}$$

yielding $V_a = 0.442$ V

2. For the *p-n* Si junction from the previous problem calculate the width of the space charge region for the applied voltages V = −10, 0, and +0.3 V. $\varepsilon_{Si} = 11.9$

Solution: (3.2)

Taking into account that at room temperature all donors and acceptors are ionized, i.e. $n = N_d$ and $p = N_a$, from the values found in the previous problem and

$$\omega = \left(\frac{\epsilon(\varphi_d - V)}{2\,\pi\ e}\ \frac{N_d + N_a}{N_d N_a}\right)^{1/2}$$

we get ω(−10V) = 2 μm , ω(0V) = 0.5 μm , and ω(+0.3V) = 0.4 μm.

3. For the parameters given in the previous problem find the maximum electric field within the space charge region. Compare these values with the electric field within a shallow donor: $E \approx e/\varepsilon_{Si} a^2_B$, where a_B is the Bohr radius of a shallow donor, $a_B = \varepsilon_{Si} h^2 / m^*_e e^2$ and $m_e / m_o = 0.33$.

Solution: (3.3)

$$E = 2\left(\frac{2\,\pi\ e\,(\varphi_d - V)}{\epsilon}\ \frac{N_d\,N_a}{N_d + N_a}\right)^{1/2}$$

we obtain that E(−10V) = 10^5 V/cm, E(0V) = 2.6×10^4 V/cm, and E(+0.3V) = 2×10^4 V/cm.

The electric field within a shallow donor is, in turn, $E \approx 3.4 \times 10^5$ V/cm, that is, comparable to that of the *p-n* junction.

4. Calculate the capacity of the *p-n* junction from the problem 2 if the area of the junction is 0.1 cm^2.

Solution: (3.4)

$$C = \frac{\epsilon\, S}{4\,\pi\,\omega}\,.$$

we get C(−10V) = 0.5 nF, C(0V) = 2 nF, and C(+0.3V) = 2.6 nF.

5. n-Si of a *p-n* Si junction has a resistivity of 1 Ώcm. What should be the resistivity? of p-Si so that 99 % of the total width of the space charge region would be located in n-Si (p$^+$-n junction)? For the parameters needed see problem 1.

Solution: (3.5)

From the conditions of the problem $\omega_a = 0.01\omega$ and $\omega_d = 0.99\omega$. Since $\omega_a/\omega_d = N_d/N_a$, we get that $N_a = 99\ N_d$. Because $N_d = 1/e\rho\mu_n = 4.5 \times 10^{15}$ cm^{-3}, we get $N_a = 4.4 \times 10^{17}$ cm^{-3}.

Solution: (4.5)

From the parameters given, we find $\lambda_n = 2.5 \times 10^{-6}$ cm and $\lambda_p = 8.5 \times 10^{-7}$ cm. If $N_d = N_a = N$ the width of the space charge region is

$$w = \left(\frac{\epsilon\, \varphi_d}{\pi\, e\, N}\right)^{1/2}$$

By definition, w = λ. Substituting

$$\varphi_d = \frac{KT}{e}\, ln\left(\frac{N^2}{n_i^2}\right)$$

into the expression above and after some simplifications, we get

$$N = \frac{\epsilon}{\pi\, e\lambda^2}\,\frac{2\,KT}{e}\, ln\left(\frac{N}{n_i}\right)$$

Solving the above equation numerically, we obtain N = 6.8 ×

10^{18} cm^{-3} and 6.2 × 10^{19} cm^{-3} for electron and holes, respectively.

6. When a silicon p_{+}-n junction is reverse-biased to 30 V, the depletion-layer capacitance is 1.75 nF/cm^2. If the maximum electric field at avalanche breakdown is 3 × 10^5 V/cm, find the breakdown voltage. $\varepsilon_{Si} = 11.9$.

Solution: (4.6)

Since $C = \epsilon / 4 \pi w_0$, and under strong reverse bias $w_0 \approx (\epsilon V / 2 \pi e N_d)^{1/2}$, we obtain $N_d = 1.1 \times 10^{15}$ cm^{-3}. Maximum electric field is at the interface and for a p$^+$-n junction equals $E \approx 4 \pi e N_d w_1 / \epsilon$. From conditions of the problem we find that at the breakdown w_1 = 18 μm and, hence, the breakdown voltage is 273 V .

7. For a p$^+$-n Si junction with $N_d = 10^{16}$ cm^{-3}, the breakdown voltage is 32 V. Calculate the maximum electric field at the breakdown. $\varepsilon_{Si} = 11.9$.

Solution: (4.7)

The width of the space charge region is $w \approx (\epsilon V / 2\pi e N_d)^{1/2}$= 2 μm. From here we get that the maximum electric field at the breakdown is

$$E = \frac{4\pi e N d}{\epsilon} w \approx 3 \text{ x } 10^5 \; V/_{cm}$$

Problem:2

A Si p-n junction has dopant concentrations $N_D = 2 \text{ x } 10^{15}$ cm^{-3} and $N_A = 2 \text{ x } 10^{16}$ cm^{-3}.

Calculate the built-in potential V_{bi} in eV and the total width of the depletion region $W = x_{n0} + x_{p0}$ at zero bias (that is, $V_a = 0$) and under a reverse bias V_a = - 8V.

Solution:

(a) Given $n_i = 1{:}5 \text{ x } 10^{10}$ cm^{-3}, the build-in potential is:

$$V_{bi} = \frac{KT}{q}\ln\frac{N_A N_D}{n_i^2} = \frac{KT}{q}\ln\frac{p_p}{p_n} = 0.671\ eV$$

(b) The depletion width with a bias V_a is give by:

$$W = \sqrt{\frac{2\,\epsilon_o\,(V_{bi} - V_a)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

For V_a = 0V , we have W = 0*:*691*μm* and for V_a = - 8V , we have W = 2*:*475 *μm*.

Problem: 3

A Si p-n junction is reverse-biased with V_a = -10V . Determine the percent change in junction (depletion) capacitance and built-in potential if the doping in the p region is increased by a factor of 2.

Solution:

Let us assume the N_A and N_D values are N_D = 2 *x* 10^{15} cm^{-3} and N_A = 2 *x* 10^{16} cm^{-3} , then the build-in potential is calculated by the equation, we have:

$$V_{bi} = \frac{KT}{q}\ln\frac{N_A N_D}{n_i^2} = \frac{KT}{q}\ln\frac{p_p}{p_n} = 0.671\ eV$$

and

$$V_{bi} = \frac{KT}{q}\ln\left(\frac{2\ N_A N_D}{n_i^2}\right) = 0\!:6888\ eV$$

Therefore, V_{bi} is increased by 2*:*67%.

The depletion capacitance can be calculated by

$$C_j = \frac{\epsilon\ A}{W}$$

Where A is the cross-sectional area of the junction and W is the depletion width calculated by:

$W = \sqrt{\frac{2\,\epsilon_o\,(V_{bi} - V_a)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$ = 2*:*7549 µ*m*

$W = \sqrt{\frac{2\,\epsilon_o\,(V_{bi} - V_a)}{q}\left(\frac{1}{2\ N_A} + \frac{1}{N_D}\right)}$ = 2*:*6938 µ*m*

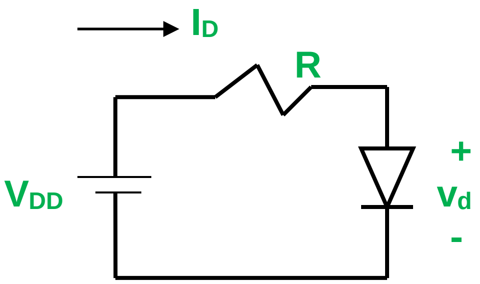
Therefore, the percent change in junction depletion capacitance is given by:

$$\frac{C_j' - C_j}{C_j} = \frac{w - w^{\cdot}}{w^{\cdot}} = 2.27\ \%$$

Switching time

Example 1.8

Determine the current I_D and the diode voltage V_D for the circuit shown with V_{DD}=5V and R=1 K . Assume that the diode has a current of 1 mA at a voltage of 0.7V and that its voltage drop changes by 0.1V for every decade change in current. To begin the iteration, we assume that V_D = 0.7 V and use the next equation to determine the current.



Assuming that V_{DD} is greater than 0.5 V or so, the diode current will be much greater than I_s, and we can represent the diode i-v characteristic by the exponential relationship, resulting in

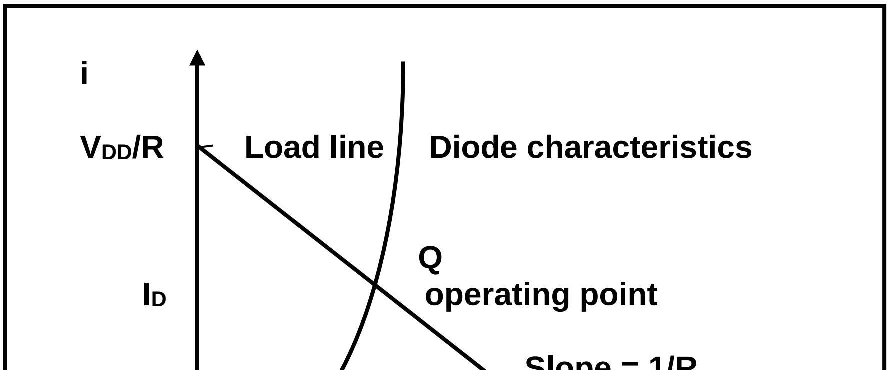
$$I_D = I_s\, e^{V_D/\eta V_T}$$

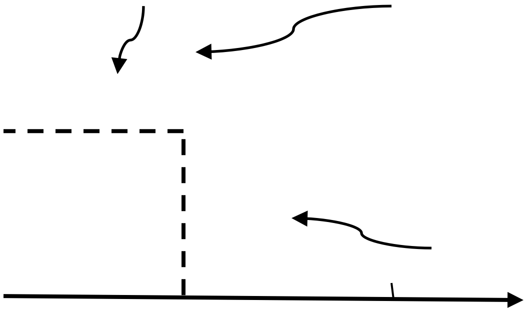
The other equation that governs circuit operation is obtained by writing a Kirchhoff loop equation, resulting in

$$I_D \frac{V_{DD} - v_D}{R}$$

Assuming that the diode parameters I_s and n are known , the two equations above has two unknown quantities I_D and V_D. Two alternative ways for obtaining the solution are graphical analysis and iterative analysis.

Graphical Analysis Using the Exponential Model

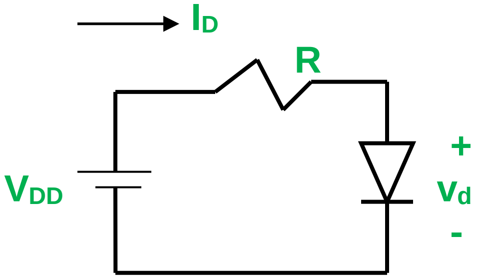




Graphical analysis is performed by plotting the relationships of the used equations above on the i-v plane. The solution can then be obtained as the coordinates of the point of intersection of the two graphs. A sketch of the graphical construction is shown. The curve represents the exponential diode equation, and the straight line is known as the load line. The load line intersects the diode curve at point Q, which represents the operating point of the circuit. Its coordinates give the values of I_D and V_D.

Example 1.8

Determine the current I_D and the diode voltage V_D for the circuit shown



With V_{DD}=5V and R = 1 K . Assume that the diode has a current of 1 mA at a voltage of 0.7V and that its voltage drop changes by 0.1V for every decade change in current. To begin the iteration, we assume that V_D = 0.7 V and determine the current

$$I_D \frac{V_{DD} - v_D}{R} = \frac{5 - 0.7}{1} = 4.3\ mA$$

We then use the diode equation to obtain a better estimate for V_D. This can be done by employing Eq. as

$$V_2 - V_1 = 23\,\eta\,V_T\,log\ {}^{i_1}/_{i_2}$$

$$23\,\eta\, V_T = 0.1\,V\;, thus\; V_2 = \;V_1 + \;0.1\, log\, {}^{i_1}/_{i_2}$$

Substituting $V_1 = 0.7$ V, $I_1 = 1$ mA, and $I_2 = 4.3$ mA results in $V_2 = 0.763$ V. Thus the results of the first iteration are $I_D = 4.3$ mA and $V_D = 0.763$ V. The second iteration proceeds in a similar manner:

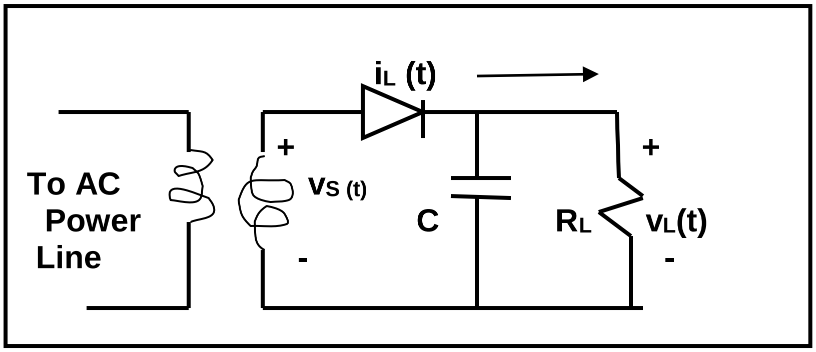
$$I_D\; \frac{V_{DD} - \;v_D}{R} = \frac{5 - 0.763}{1} = 4.237\; mA$$

$$V_2 = 0.763 + 0.1\; log\; \frac{4.237}{4.3} = 0.762\; V$$

Thus the second iteration yields $I_D = 4.237$ mA and $V_D = 0.762$ V. Since these values are not much different from the values obtained after the first iteration, no further iterations are necessary, and the solution is $I_D = 4.237$ mA and $V_D = 0.762$ V.

Example 11

The dc power supply circuit of figure shown is to be designed for the following specifications: dc output voltage = +15 V, dc load current = 100 mA, and percent ripple = 5%. The rectifier diode can be modeled with the parameters $V_{D0} = 0.7$ V and $R_D = 0$. Calculate the required transformer secondary ac rms voltage and the value of the filter capacitor. Assume a frequency f = 60 Hz.



Solution.

The peak secondary transformer voltage is given by $V_1 = 15+0.7 = 15.7$ V. The ac rms voltage is 15.7/ √ 2 = 11.1 V rms. The effective value of the load resistor is $R_L = 15 / 0.1 = 150\ \Omega$. The value of C can be calculated form equation $Percent\ Ripple\; \le \left[1 - exp\left(\frac{-T}{R_L\,C}\right)\right]\; x\,100\,\%$

Where equality used. We obtain

$$C = \frac{T}{R_L}\frac{-1}{ln\left(1-\frac{\%\,ripple}{100}\right)} = \frac{1/50}{150}\,\frac{-1}{ln\left(1-5/100\right)} = 2170\ \mu F$$

If follows from the inequality equation $Percent\ Ripple \le \left[1 - exp\left(\frac{-T}{R_L\,C}\right)\right] x\,100\,\%$ that this value for C gives a percent ripple that is less than 5%.

Part (15) Solved PROBLEMS
photoelectric effect

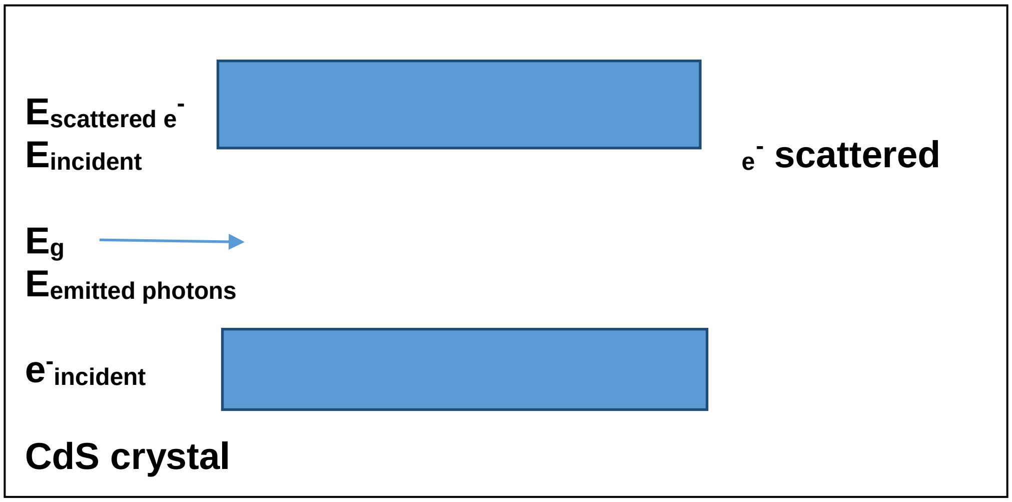
Problem (1)

(a) An electron beam strikes a crystal of cadmium sulfide (CdS). Electrons scattered by the crystal move at a velocity of 4.4 x 10^5 m/s. Calculate the energy of the incident beam. Express your result in eV. CdS is a semiconductor with a band gap, Eg, of 2.45 eV.

(b) Cadmium telluride (CdTe) is also a semiconductor. Do you expect the band gap of this material to be greater or less than the band gap of CdS? Explain.

Solution

(a)



$$E_{incident\ e-} = E_{emitted\ v} + E_{scattered\ e-} = E_g + \frac{m\,v^2}{2}$$

$$E_{incident\ e-} = 2.45\ eV + \frac{1}{2}\ x\ \frac{9.11\ x\ 10^{-31}\ kg\ x\ \left(4.4\ x\ 10^5\,\frac{m}{s}\right)^2}{1.6\ x\ 10^{-19}\,\frac{eV}{J}}$$

$$= 2.45\ eV + 0.55\ eV = 3\ eV$$

(b) E_g(CdTe) <E_g(CdS)

The Cd–S bond is stronger than Cd–Te bond because although both S and Te are group 16, Te is much larger than S.

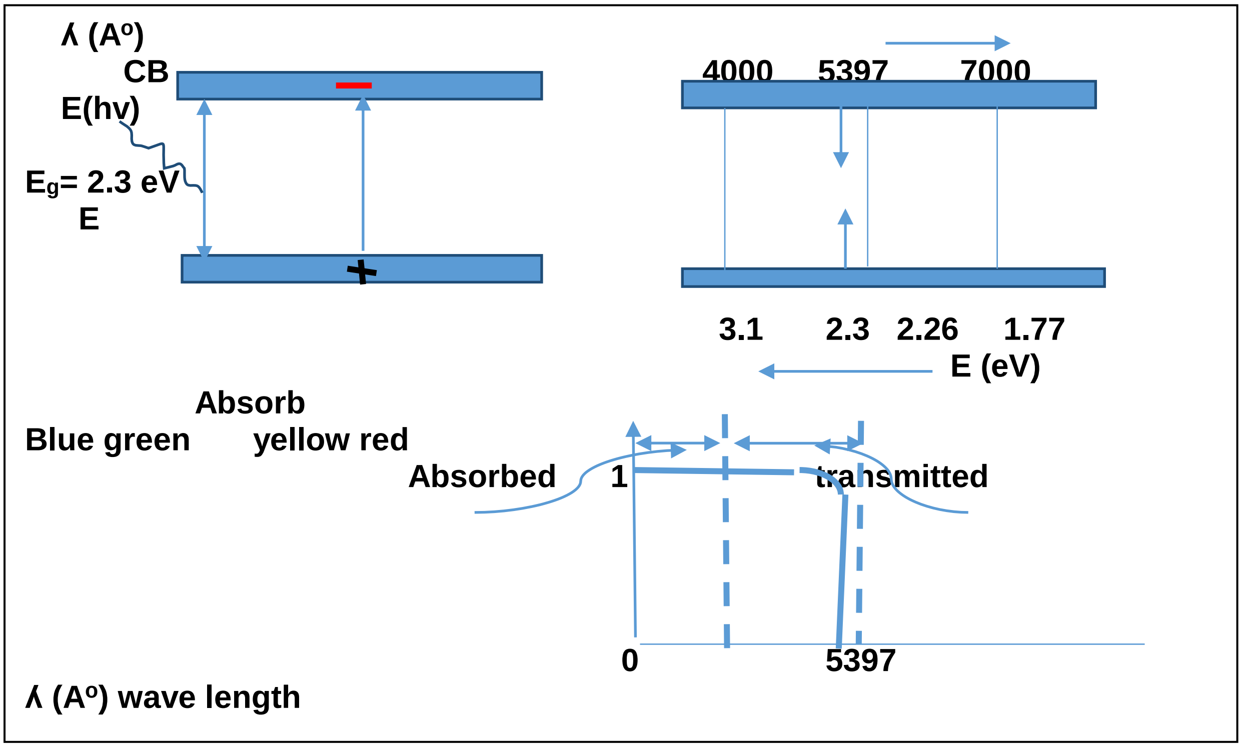
Problem #8

The energy gap (E_g) of ZnSe is 2.3 eV.

(a) Is this material transparent to visible radiation? Substantiate your answer.

(b) How could you increase the electrical conductivity of this material? Give the reasons for the effectiveness of your suggested approach.

Solution



(a) The optical properties of ZnSe can be explained when comparing the energy band of the visible spectrum with the energy band diagram of ZnSe. Absorption takes place via photo excitation for all radiation with E ≥ 2.3 eV. From the energy distribution of the visible spectrum we recognize that the blue– green portion has photon energies in excess of the band gap (E_g = 2.3eV) and thus will be absorbed. The yellow–red potion, on the other hand, has photon energies less than the band gap – it will be transmitted. ZnSe, therefore, is expected to exhibit a yellowish–red color.

(b) In principle there are two ways to increase the electrical conductivity of ZnSe:

a. A temperature rise. Any rise in temperature will increase the number of “thermally activated” charge carriers in the conduction band (electrons) and in the valence band (holes) and, thus, the electronic conductivity.

Aside: The electrical conductivity of solids, demonstrated by the flow of electronic charge carriers under an applied electric field (E), can be

formulated through Ohm's law, J = σE, which states that the current density (J = number of charges transported through a unit area in a unit time) is proportional (σ = conductivity) to the applied electric field. Accordingly:

$$J = N\, e\, v_d$$

where N = number of charge carriers/unit volume, e = electronic charge and v_d = average drift velocity of charge carriers in an applied electric field. We thus obtain:

$$\sigma = \frac{N\, e\, v_d}{E}$$

and if we define d(v /E) = μ, the charge carrier mobility, we have:

$$\sigma = N\, e\, \mu$$

In intrinsic semiconductors we have both electrons and holes contributing to conduction:

e e n h e h

$$\sigma = N_e\, e\, \mu_e + N_n e \mu_h = Ne(\mu_e + \mu_h)$$

since $N_e = N_h$. Taking the number of thermally generated charge carriers, given by the relationship

$$N = AT^{3/2}\, e^{-E_g/2KT}$$

we obtain the temperature dependence of the conductivity as:

$$\sigma = AT^{3/2}\, e^{-E_g/2KT}\, x\, \mathrm{e}\, (\mu_e + \mu_h)$$

To assess the temperature dependence of electrical conductivity we must take into consideration that, because of increased vibration of the atoms about their lattice positions, the charge carrier mobility will decrease (increased scattering of charge carriers) with increasing temperature. This effect explains why the electronic conductivity in metals, where N is constant, will decrease with increasing temperature. In semiconductors, where N increases with temperature, the accompanying mobility effect is not apparent at low temperatures (conductivity increases), but becomes pronounced at high temperatures (conductivity decreases).

b. Introduction of shallow impurity (or defect) states close to the conduction or valence band. This is accomplished by the incorporation of appropriate dopant elements into the crystal matrix. If these impurities are shallow (~0.01 eV from the conduction or valence band), they will be totally ionized at room temperature and each will contribute an electron (donor dopant: K, N_a) or holes

(acceptor dopant: G, Br), thus increasing the electrical conductivity without the necessity of a temperature rise. (Be aware that certain defects in the crystal lattice may also increase the electronic conductivity.)

9. Electron mobility in Si is 1400 cm^2 V^{-1}s^{-1}. Calculate the mean free time in scattering (Relaxations time) of electrons.
Effective mass is $m^*_e/m_0 = 0.33$.
Solution:
From $\mu = e\tau/m*$ we get that $\tau = 2.6\times10^{-13}$ s.

Example 1.1

A metal has a work function of 4.3 V. What is the minimum photon energy in Joule to emit an electron from this metal through the photo-electric effect?
What are the photon frequency in Terahertz and the photon wavelength in micrometer?
What is the corresponding photon momentum? What is the velocity of a free electron with the same momentum?

Solution

The minimum photon energy, E_{ph}, equals the work function, Φ_M, in units of electron volt or 4.3 eV. This also equals:

$$E_{ph} = q\,\phi_m = 1.6\ x\ 10^{-19}\ x\ 4.3 = 6.89\ x\ 10^{-19}$$

The corresponding photon frequency is:

$$f_{ph} = \frac{6.89\ x\ 10^{-19}}{6.626\ x\ 10^{-34}} = 1040\ THz$$

The corresponding wavelength equals:

$$\lambda = \frac{h\,c}{E_{ph}} = \frac{6.626\ x\ 10^{-34}\ x\ 3\ x\ 10^{8}}{6.89\ x\ 10^{-19}} = \frac{1.24\ \mu}{E_{ph}\ (eV)} = 0.288\ \mu m$$

The photon momentum, p, is:

$$p = \frac{h}{\lambda} = \frac{6.626\ x\ 10^{-34}}{0.288\ x\ 10^{-6}} = 2.297\ x\ 10^{-27}\ kg\ m/s$$

And the velocity, v, of a free electron with the same momentum equals:

$$v = \frac{p}{m_o} = \frac{2.297\ x\ 10^{-27}}{9.11\ x\ 10^{-31}} = 2522\ m/s$$

Where m_o is the free electron mass

$$E_{ph} = q\ \phi_m = 1.6\ x\ 10^{-19}\ x\ 4.3 = 6.89\ x\ 10^{-19}$$

Example 1.2 (the blackbody radiation)

The spectral density of the sun peaks at a wavelength of 900 nm. If the sun behaves as a black body, what is the temperature of the sun?

Solution:

A wavelength of 900 nm corresponds to a photon energy of:

$$E_{ph} = \frac{h\ c}{\lambda} = \frac{6.626\ x\ 10^{-34}\ x\ 3\ x\ 10^{8}}{900\ x\ 10^{-9}} = 2.21\ x\ 10^{-19} Joule$$

Since the peak of the spectral density occurs at 2.82 kT, the corresponding temperature equals:

$$T = \frac{E_{ph}}{2.82\ k} = \frac{2.21\ x\ 10^{-19}}{2.82\ x\ 1.38\ x10^{-23}} = 5672\ Kelvin$$

Example 1.3

An electron is confined to a 1 micron thin layer of silicon. Assuming that the semiconductor can be adequately described by a one-dimensional quantum well with infinite walls, calculate the lowest possible energy within the material in units of electron volt. If the energy is interpreted as the kinetic energy of the electron, what is the corresponding electron velocity? (The effective mass of electrons in silicon is 0.26 m_0, where $m_0 = 9.11 \times 10^{-31}$ kg is the free electron rest mass).

Solution:

The lowest energy in the quantum equals:

$$E_1 = \frac{h^2}{2\,m^*}\left(\frac{1}{2\,L_p}\right)^2 = \frac{(6.626\,x\,10^{-34})^2}{2\,x\,0.26\,x\,9.11\,x\,10^{-31}}\left(\frac{1}{2\,x\,10^{-6}}\right)^2$$

$$= 2.32x\,10^{-25} joule = 1.45\,meV$$

Then velocity of an electron with this energy equal:

$$\nu = \sqrt{\frac{2E_1}{m^*}} = \sqrt{\frac{2\,x\,2.32\,x\,10^{-25}}{0.26\,x\,9.11\,x\,10^{-31}}} = 1.399\,km/s$$

Example 1.4

Consider an infinitely long cylinder with charge density r, dielectric constant ε₀ and radius r₀. What is the electric field in and around the cylinder?

Solution:

Because of the cylinder symmetry one expects the electric field to be only dependent on the radius, r. Applying Gauss's law one finds:

$$\vec{\varepsilon}\vec{A} = \varepsilon\,2\,\pi\,r\,L = \frac{Q}{\varepsilon_0} = \frac{\rho\,\pi\,r^2\,L}{\varepsilon_o}$$

And

$$\vec{\varepsilon}\vec{A} = \varepsilon\,2\,\pi\,r\,L = \frac{Q}{\varepsilon_0} = \frac{\rho\,\pi\,r_o^2\,L}{\varepsilon_o}$$

Where a cylinder with length L was chosen to define the surface A and edges effects were ignored. the electric field then equal:

$$\varepsilon(r) = \frac{\rho\,r}{2\,\varepsilon_o}$$

The electric field increases with the cylinder with increasing radius. The electric field decreases outside with the cylinder with increasing radius.

Problem 1.1

Calculate the wavelength of a photon with a photon energy of 2 eV. Also, calculate the wavelength of an electron with a kinetic energy of 2 eV.

Solution The wavelength of a 2 eV photon equals:

$$\mathbf{deBroglie}\boldsymbol{l_w} = \frac{\boldsymbol{h\,C}}{\boldsymbol{E_{ph}}} = \frac{\mathbf{6.626}\ \boldsymbol{x}\ \mathbf{1}\boldsymbol{o}^{-\mathbf{34}}\ \boldsymbol{J}\ \boldsymbol{x}\ \mathbf{3}\ \boldsymbol{x}\ \mathbf{10^{8}}\ \boldsymbol{m/sec}}{\mathbf{1.602}\ \boldsymbol{x}\ \mathbf{10^{-19}}\ {}^{\boldsymbol{C}}\!/_{\mathbf{2}\ \boldsymbol{eV}}}$$

$$= \mathbf{0.62}\ \boldsymbol{\mu m}$$

where the photon energy (2 eV) was first converted to Joules by multiplying with the electronic charge. The wavelength of an electron with a kinetic energy of 2 eV is obtained by calculating the deBroglie wavelength:

$$\mathbf{deBroglie}\boldsymbol{l_w} = \frac{\boldsymbol{h}}{\boldsymbol{p}} = \frac{\mathbf{6.626}\ \boldsymbol{x}\ \mathbf{1}\boldsymbol{o}^{-\mathbf{34}}\ \boldsymbol{J}}{\mathbf{7.62}\ \boldsymbol{x}\ \mathbf{10^{-25}}\ \boldsymbol{kg}\ {}^{\boldsymbol{m}}\!/_{\boldsymbol{s}}} = \mathbf{0.87}\boldsymbol{nm}$$

Where the momentum of the particle was calculated from the kinetic energy:

$$\boldsymbol{p} = \sqrt{\mathbf{2}\ \boldsymbol{m\,E}} = \sqrt{\mathbf{2}\ \boldsymbol{x}\ \mathbf{9.11}\ \boldsymbol{x}\ \mathbf{1}\boldsymbol{o}^{-\mathbf{31}}\ \boldsymbol{kg}\ \boldsymbol{x}\ \mathbf{1.6}\ \boldsymbol{x}\ \mathbf{10^{-19}}\ \boldsymbol{c}\ \boldsymbol{x}\ \mathbf{2}\ \boldsymbol{eV}}$$

$$= \mathbf{7.64}\ \boldsymbol{x}\ \mathbf{10^{-25}}\ \boldsymbol{kg\ m/s}$$

Problem 1.2

Consider a beam of light with a power of 1 Watt and a wavelength of 800 nm. Calculate

a) the photon energy of the photons in the beam,

b) the frequency of the light wave and

c) the number of photons provided by the beam in one second. Solution

The photon energy is calculated from the wavelength as:

$$\boldsymbol{E_{ph}} = \frac{\boldsymbol{h\,C}}{\mathbf{deBroglie}\boldsymbol{l_w}} = \frac{\mathbf{6.626}\ \boldsymbol{x}\ \mathbf{1}\boldsymbol{o}^{-\mathbf{34}}\ \boldsymbol{Js}\ \ \boldsymbol{x}\ \mathbf{3}\ \boldsymbol{x}\ \mathbf{10^{8}}\ \boldsymbol{m/s}}{\mathbf{800}\ \boldsymbol{x}\ \mathbf{10^{-9}}\ \boldsymbol{m}}$$

$$= \mathbf{2.48}\ \boldsymbol{x}\ \mathbf{10^{-19}}\boldsymbol{J}$$

or in electron Volt:

$$E_{ph} = \frac{2.48x\ 1o^{-19}\ J}{1.6\ x\ 1o^{-19}c} = 1.55eV$$

The frequency then equals: -

$$n = \frac{E_{ph}}{h} = \frac{2.48\ x\ 10^{-19}J}{6.626\ x\ 1o^{-34}\ Js} = 375\ THz$$

And the number of photons equals the ratio of the optical power and the energy per photon:

$$\#\ photons = \frac{1\ watt}{E_{ph}} = \frac{1\ watt}{2.48\ x\ 10^{-19}J} = 4\ x\ 10^{18}$$

Problem 1.3

Show that the spectral density, u_w(equation 1.2.4) peaks at E_{ph} = 2.82 kT. Note that a numeric iteration is required.

Solution

The spectral density, uw, can be rewritten as a function of

$$\chi = \frac{\hbar\ w}{KT}$$

$$u_w = \frac{K^3T^3}{\hbar^2p^2c^2}\frac{\chi^3}{exp\ (\chi) - 1}$$

The maximum of this function is obtained if its derivative is zero or:

$$\frac{du_w}{d\chi} = \frac{3\ \chi^2}{exp\ (\chi) - 1} - \frac{\chi^2 exp\ (\chi)}{(exp\ (\chi) - 1)^2} = 0$$

Therefore χ must satisfy:

$$3 - 3\ exp\ (-\ \chi) = \chi$$

This transcendental equation can be solved starting with an arbitrary positive value of χ. A repeated calculation of the left hand side using this value and the resulting new value for χ quickly converges to χ_{max} = 2.82144. The maximum spectral density therefore occurs at:

$$E_{ph\ max} = \ \chi_{max}\ KT = \ 2.82144\ \text{KT}$$

Problem 1.4

Calculate the peak wavelength of blackbody radiation emitted from a human body at a temperature of 37°C.

Solution The peak wavelength is obtained through the peak energy

$$\mathbf{deBrogliel}_w = \frac{h\,C}{E_{ph\,max}} = \frac{h\,C}{2.82\;KT}$$

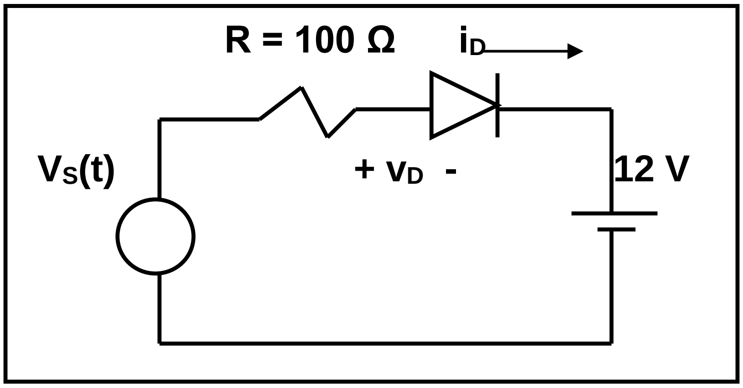
$$= \frac{6.626\;x\;10^{-34}\;x\;3\;x\;10^{8}}{2.82\;x\;1.38\;x\;10^{-23}\;x\;310.15} = 1.65\;x\;\;10^{-5}\;\mu m$$

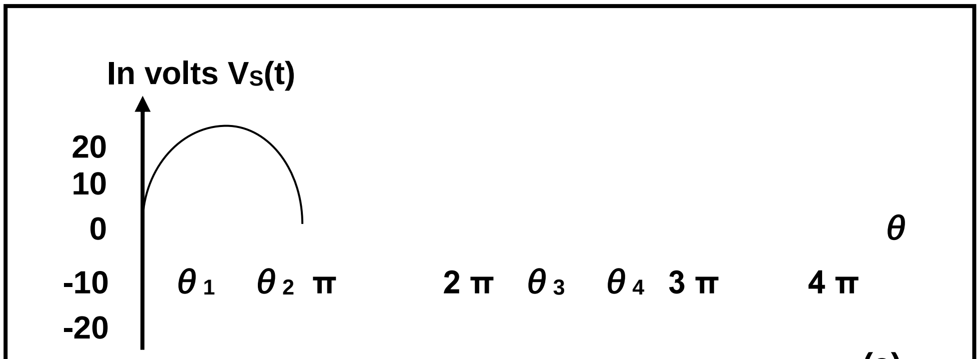
Where the temperature was first converted to units Kelvin.

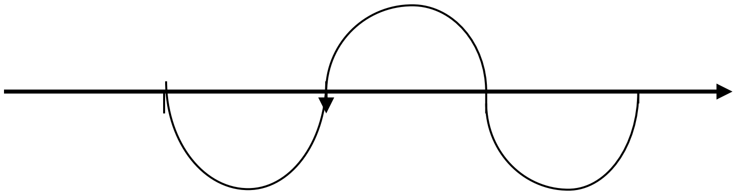
Small signal

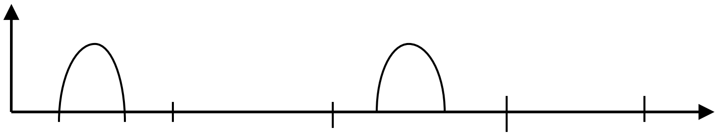
Example

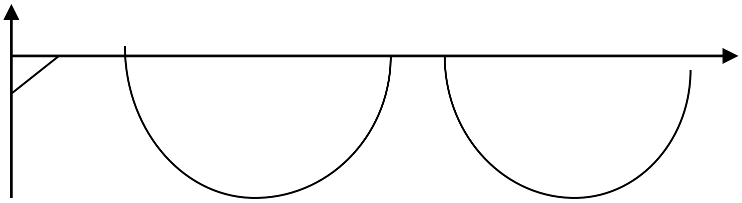
In the circuit shown, a sinusoidal signal voltage is used to charge a 12-V car battery. The source voltage has amplitude of 20 V that is shown in Fig. 2.1.5(a). Find the values of i_D and v_D and sketch their waveforms. Also, find the average value of the current charging the battery.











Solution

If the diode is forward-biased, $v_D = 0$, and the diode current is

$$\mathbf{i_D = \frac{(v_s - 12)}{100}\ A}$$

It is convenient to describe the time-varying source voltage as

$$v_S(\theta) = 20 \sin(\theta),$$

Where $\theta = (2\pi f t)$ and f is the frequency. To have $i_D > 0$ (forward-biased), v_S must be greater than 12 V; i.e. only if $v_S > 12$ V, there is a current in the loop. Otherwise the diode will be reverse-biased and will not conduct. Thus, the sketch of the diode current is shown in Fig. (b). Clearly, the diode conducts between θ_1 and θ_2 during the first cycle of input sinusoid, and again between θ_3 and θ_4 during the second cycle.

If $i_D > 0$, $v_D = 0$. If $v_S < 12$ V, the diode is reverse-biased, and i_D must be equal to zero. Therefore, if $v_S < 12$ V, KVL gives

$$\mathbf{v_D = (v_s - 12)V}$$

The sketch of the waveform for v_D is shown in (c). The average current

can be found by finding the average value of the current during one period.

First, during the first period, we observe that

$$i_D = \frac{(v_s - 12)}{100}\ A\,, \quad \theta_1 \le \theta \le \theta_2 \quad = 0$$

Otherwise,

Since $v_S(\theta) = 20\sin(\theta)$, the values of the angles can be found using

$$\theta_1 = \sin^{-1}\left(\frac{3}{5}\right) \text{ and } \theta_2 = \pi - \text{son}^{-1}\left(\frac{3}{5}\right) = \pi - \theta_1$$

The average value of the current is

$$I_D = \frac{1}{2\,\pi}\int_{\theta_1}^{\pi-\theta_1} \frac{20\sin\theta - 12}{100}\, d\theta = \frac{0.4\cos\theta_1 - 0.12}{2\,\pi}\ A$$

Substituting the value of θ_1 in the above equation, we find that

I_D = 15.51 mA.

Breakdown

Example

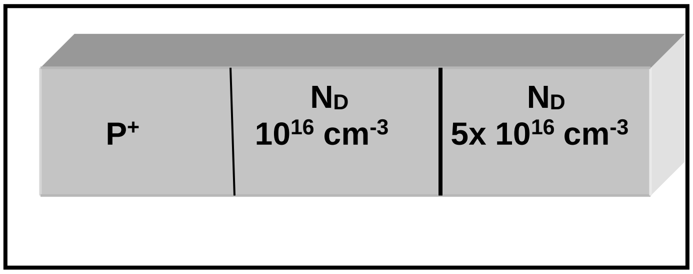
An abrupt pn junction has doping densities of $N_A = 5\text{x}10^{15}$ atoms/cm^3 and $N_D = 10^{16}$ atoms/cm^3. Calculate the breakdown voltage if $E_{crit} = 3\text{x}10^5$ V/cm.

Solution

$$V_R = \frac{\varepsilon_{si}\,(N_A + N_D)}{2\,q\,N_A N_D} E_{max}^2 = \frac{1.04\ x\ 10^{-12}\ (5\ x\ 10^{15} + 10^{16})}{2\ x\ 1.6\ x\ 10^{-19}\ x\ 5\ x\ 10^{15}\ x\ 10^{16}} = 88\ V$$

Example

Determine breakdown voltage for a pn junction shown , assume V_{bi} = 0.9 V

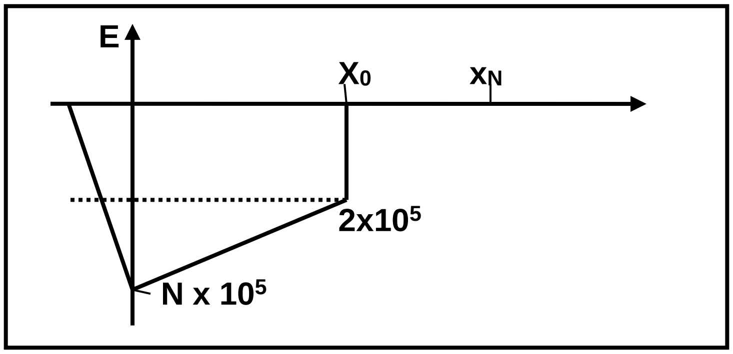


Solution:

First perform a check whether the depletion width still lies in the highly doped region or nit. If it does, then

$$W = 4\ x\ 10^5\ x\ \frac{\varepsilon_s}{q\ N_D} = 2.6\ \mu m$$

This shows that the depletion width extend into the highest doped N-region as well resulting in the following diagram.



The electric field at x_0 is obtained using the expression

$$W = 4\ x\ 10^5\ x\ \frac{\varepsilon_s}{q\ N_{D1}}\ x\ 1.3\ \mu m$$

Using the Poisson's equation

$$x_N - x_D = \frac{2\ x\ 10^5}{\left(\frac{q}{\varepsilon_s\ N_{D2}}\right)} = 0.26\ \mu m$$

The area under the electric field curve will be equal to V_{bi} + BV , so that , BV = 40.7 V

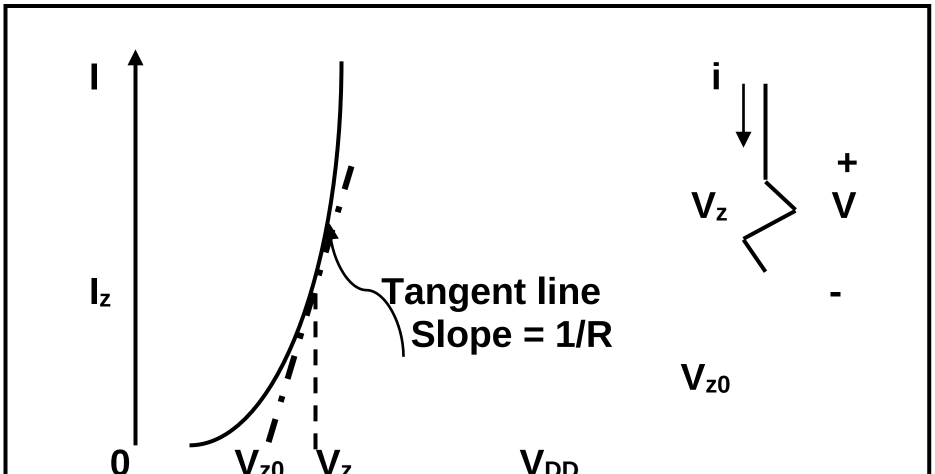
Example 6

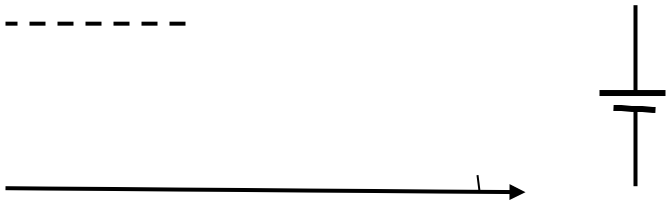
A Zener diode at room temperature (V_T = 0.0259 V) has the specifications V_Z = 10 V, I_Z = 10 mA , and r_z = 20 Ω. Calculate

(a) the Zener breakdown ideality factor η,

(b) the voltage V_{Z0} in the linear circuit shown, and

(c) the voltage at which the breakdown current is I_Z / 10.





Solution.

(a) Equation $r_z = {}^{\eta_z V_T}/_{I_z}$ be used to solve for ηz to obtain

$$\eta_z = \frac{I_z r_z}{V_T} = \frac{10\,m\,x\,20}{25.9\,m} = 7.72$$

(b) The voltage V_{Z0} is calculated from Eq. $V_{z0} = V_z - I_z r_z$ to obtain

$$V_{z0} = V_z - I_z r_z = 10 - 10\,m\,x\,20 = 9.8\,V$$

(c) If $I_S << I_Z$, Eq.

$$i = I_s \left[exp\left(\frac{V}{\eta\,V_T}\right) - 1\right] - I_z\,exp\left(-\frac{v + V_z}{\eta_z V_T}\right)$$

Can be solved for the voltage at which i = −1 mA as follows:

$$v = -V_z - \eta_z V_T\,ln\left(\frac{i}{-I_z}\right) = -10 - 7.72\,x\,25.9\,m\,x\,ln\,(0.1)$$
$$= -9.54\,V$$

The reverse-bias voltage is the negative of this, i.e. +9.54 V.

Part (16) Solved PROBLEMS

Schottky Diode

Problem (1)

Find a height of the potential barrier for a Au-n-Ge Schottky contact at room temperature (T = 293 K) if ρ = 1 Ωcm, $\psi_{Au} = 5.1$ eV, and $\chi_{Ge} = 4.0$ eV. Electron mobility in Ge is 3900 $cm^2\,V^{-1}\,s^{-1}$, density of the states in the conduction band is $N_c = 1.98\times10^{15} \times T^{3/2}\,cm^{-3}$.

Solution:

$eV_d = 0.88$ eV

Problem (2)

Calculate the depletion width for a Pt-n-Si Schottky diode (T = 300 K) at V = 0, +0.4, and −2 V. Concentration of doping impurity in Si equals 4×10^{16} cm^{-3}. Work function of Pt is 5.65 eV, electron affinity of Si is 4.05 eV, ϵ_{Si} = 11.9, density of the states in the conduction band is $N_c = 6.2\times10^{15}\times T^{3/2}$ cm^{-3}.

Solution:

w = 0.22, 0.19, and 0.34 μm for V = 0, +0.4, and −2 V, respectively

Problem (3)

For a Schottky contact Au-GaAs calculate the maximum electric field within the space charge region at V = 0, +0.3, and −100 V. $N_d = 10^{16}$ cm^{-3}, χ_{GaAs} = 4.07 eV, ϵ_{GaAs} = 12.9. Work function of Au is 5.1 eV, T = 300 K, density of the states in the conduction band is $N_c = 8.63\times10^{13}\times T^{3/2}$ cm^{-3}.

Solution:

E = 5.1×10^4, 4.2×10^4, and 5.1×10^5 V/cm for V = 0, +0.3, and −100 V, respectively.

Problem (4)

What is the electric field E for a Schottky diode Au-n-Si at V = −5 V at the distance of 1.2 μm from the interface at room temperature if ρ = 10Ωcm, μ_n = 1400 $cm^2\ V^{-1}\ s^{-1}$, $N_c = 6.2\times10^{15}\times T^{3/2}$ cm^{-3}.

Solution:

E = 2×10^4 V/cm.

Problem (5)

Find current densities j at room temperature for a Schottky diode Pt-n-GaAs at V = +0.5 and −5 V if ρ = 50Ωcm. μ_n = 8800 $cm^2\ V^{-1}\ s^{-1}$, m_n/m_0 = 0.063, work function of Pt is 5.65 eV, χ_{GaAs} = 4.07 eV, $N_c = 8.63 \times 10^{13} \times T^{3/2}$ cm^{-3}. Apply thermionic-emission theory.

Solution:

From n = 1/eρμ_n we obtain that n = 1.4×10^{13} cm^{-3}. Thus,

$$e\,\varphi_d = \psi\, p_t - \chi_{GaAs} - KT\, ln\,\frac{N_c}{n} = 1.32\ eV$$

The average thermal velocity is

$$\boldsymbol{v_t = \left(\frac{8\,KT}{\pi\, m_n}\right)^{1/2} = 4.6\;x\;10^7\;cm/s}$$

From here we get

$$\boldsymbol{J_s = \frac{1}{4}\,e\,n\,v_T\,exp\,\left(\frac{-\,e\,\varphi_d}{KT}\right) = 3\;x\;10^{-22}\;A/cm^2}$$

From here we get

$$\boldsymbol{j = j_s\left\{exp\,\left(\frac{eV}{KT}\right) - 1\right\}}$$

we obtain j(0.5V) = 1.5×10^{-13} A/cm^2 and j(−5V) = j$_s$.

Problem (6)

The capacitance of a Au-n-GaAs Schottky diode is given by the relation 1/C^2 = 1.57×10^{15} −2.12×10^{15} V, where C is expressed in F and V is in Volts. Taking the diode area to be 0.1 cm^2, calculate the barrier height and the dopant concentration.

Solution:

φ_d= 0.74 V, n = 2.8×10^{17} cm$^{-3.}$

7. From comparison of the de Broglie wavelength of electron with the depletion width of a contact metal-n-Si, estimate the electron concentration at which Schottky diode loses its rectifying characteristics. For the estimate, assume that the height of the potential barrier a the contact is half the value of the band gap at room temperature (E_g = 1.12 eV), $m^*_e = m_0$, T = 300 K, and ϵ_{Si} = 11.9.

Solution:

A Schottky diode loses its rectifying characteristics when de Broglie wavelength, λ, of electron becomes comparable with the depletion width, ω, of the diode. Since

$$\boldsymbol{\lambda = \frac{2\,\pi\,h}{\sqrt{2\,m_o\,E}}\;and\;\omega = \frac{\sqrt{\epsilon_{si}E_g}}{4\,\pi\,e^2\,n}}$$

From the condition $\lambda \ll \omega$

We obtain that ,

$$\boldsymbol{n \ll \frac{3\,\epsilon_{si}m_o\,KT\,E_g}{16\,\pi^3 e^2 h^2}}$$

Here, we assumed that the mean energy of electron is E = 3kT/2 and

the potential barrier at the contact is φ_d = E$_g$/2e. Substituting numerical values in the above expression we get that for proper functioning of the Schottky diode, electron concentration must be significantly less than 2×10^{19} cm^{-3}.

Problem 3.1

Consider a gold-GaAs Schottky diode with a capacitance of 1 pF at -1 V. What is the doping density of the GaAs? Also calculate the depletion layer width at zero bias and the field at the surface of the semiconductor at -10 V. The area of the diode is 10^{-5} cm^{2}.

Solution The depletion layer width can be calculated from the capacitance yielding:

$$x_d = \frac{e_s\,A}{C} = \frac{13.1\;x\;8.854\;x\;10^{14}\;x\;10^{-5}}{10^{-12}} = 0.116\;\mu m$$

From this one can find the doping density:

$$N_d = \frac{2\;e_x(f_i - V_a)}{q\;x_d^2} = \frac{2\;x13.1\;x\;8.854\;x\;10^{14}\;x\;(f_i+1)}{1.6\;x\;10^{-19}\;x\;(1.16\;x\;10^{-5})^2}$$

Provided one knows the built-in potential

$$f_i = f_B - V_t\;ln\;\frac{N_c}{N_d} = 4.8 - 4.07 - 0.0259\;ln\;\frac{4.35\;x\;10^{17}}{N_d}$$

Which in turn depends on the doping density. Starting with f$_i$ = 0.7 one finds N$_d$ = 1.83 x 10^{17} cm^{-3} and the corresponding built-in potential f$_i$ = 0.708. Further iteration yields the result: N$_d$ = 1.84 x 10^{17} cm^{-3}.

The depletion layer width at zero bias equals:

$$x_d = \sqrt{\frac{2\;e_x(f_i - V_a)}{q\;N_d}} = \frac{2\;x13.1\;x\;8.854\;x\;10^{14}\;x\;(0.708-0)}{1.6\;x\;10^{-19}\;x\;1.84\;x\;\;10^{17}}$$
$$= 0.075\;\mu m$$

And the electric field at the surface for V$_a$ = -10 V equals:

$$\varepsilon\,(x=0)=\frac{q\,N_d x_d}{e_s}$$

$$=\sqrt{\frac{2\,(f_i-V_a)\,q\,N_d}{e_s}}\sqrt{\frac{2\,x\,20.7\,x\,1.6\,x\,10^{-19}\,x\,1.84\,x\,10^{17}}{13.1\,x\,8.854\,x\,10^{-14}}}=737\,KV/cm$$

Example 3.1

Consider a chrome-silicon metal-semiconductor junction with $N_d = 10^{17}$ cm^{-3}. Calculate the barrier height and the built-in potential. Repeat for a p-type semiconductor with the same doping density.

Solution The barrier height equals:

$$\phi_B=\Phi_m-\chi=4.5-4.05=0.45\,V$$

Note that this value differs from the one listed in Table , since the work function in vacuum was used.

The built-in potential equals:

$$\phi_1=\phi_B-V_t\,ln\,\frac{N_c}{N_d}=0.45-0.0259\,ln\,\frac{2.82\,x\,10^{19}}{10^{17}}=0.3\,V$$

The barrier height for the chrome/p-silicon junction equals:

$$\phi_B=\chi+\frac{E_g}{q}-\Phi_m=4.05+1.12-4.5=0.67\,V$$

And the built-in potential equals:

$$\phi_1=\phi_B-V_t\,ln\,\frac{N_v}{N_a}=0.67-0.0259\,ln\,\frac{1.83\,x\,10^{19}}{10^{17}}=0.53\,V$$

1.2 Schottky Diode

1. Find a hight of the potential barrier for a Au-n-Ge Schottky contact at room temperature (T = 293 K) if ρ = 1Ώ cm, ψ_{Au} = 5.1 eV, and χ_{Ge} = 4.0 eV. Electron mobility in Ge is 3900 cm^2 V^{-1} s^{-1}, density of the states in the conduction band is N_c = 1.98 × 10^{15} × $T^{3/2}$ cm^{-3}.

Solution: (2.1)

eV_d = 0.88 eV.

2. Calculate the depletion width for a Pt-n-Si Schottky diode ($T = 300$ K) at $V = 0$, $+0.4$, and -2 V. Concentration of doping impurity in Si equals 4×10^{16} cm^{-3}. Work function of Pt is 5.65 eV, electron affinity of Si is 4.05 eV, $\varepsilon_{Si} = 11.9$, density of the states in the conduction band is $N_c = 6.2 \times 10^{15} \times T^{3/2}$ cm^{-3}.

Solution: (2.2)
$w = 0.22$, 0.19, and 0.34 µm for $V = 0$, $+0.4$, and -2 V, respectively.

3. For a Schottky contact Au-GaAs calculate the maximum electric field within the space charge region at $V = 0$, $+0.3$, and -100 V. $N_d = 10^{16}$ cm^{-3}, $\chi_{GaAs} = 4.07$ eV, $\varepsilon_{GaAs} = 12.9$. Work function of Au is 5.1 eV, $T = 300$ K, density of the states in the Conduction band is $N_c = 8.63 \times 10^{13} \times T^{3/2}$ cm^{-3}.

Solution: (2.3)
$E = 5.1 \times 10^4$, 4.2×10^4, and 5.1×105 V/cm for $V = 0$, $+0.3$, and -100 V, respectively.

4. What is the electric field E for a Schottky diode Au-n-Si at $V = -5$ V at the distance of 1.2 µm from the interface at room temperature if $\rho = 10$ Ώcm, $\mu_n = 1400$ cm^2 V^{-1} s^{-1}, $N_c = 6.2 \times 10^{15} \times T^{3/2}$ cm^{-3}.

Solution: (2.4)
$E = 2 \times 10^4$ V/cm.

5. Find current densities j at room temperature for a Schottky diode Pt-n-GaAs at $V = +0.5$ and -5 V if $\rho = 50$ Ώcm. $\mu_n = 8800$ cm^2 V^{-1} s^{-1}, $m_n/m_0 = 0.063$, work function of Pt is 5.65 eV, $\chi_{GaAs} = 4.07$ eV, $N_c = 8.63 \times 10^{13} \times T^{3/2}$ cm^{-3}. Apply thermionic-emission theory.

Solution: (2.5)

From n = 1/eρμ$_n$ we obtain that n = 1.4 × 10^{13} cm^{-3}. Thus,

$$2\varphi_d = \psi_{pt} - \psi_{G_A A_s} - KT\, ln\, {}^{N_c}/_{n} = 1.32\ eV$$

The average thermal velocity is

$$vT = \left({}^{8KT}/_{\pi m_n}\right)^{1/2} = 4.6\ x\ 10^7\ {}^{cm}/_{s}$$

From here we get

$$J_s = \frac{1}{4}\, e\, n\, v_T\, exp\left(-\ {}^{e\,\varphi_d}/_{KT}\right) = 3\ x\ 10^{22}\ {}^{A}/_{cm^2}$$

Finally, from

$$J = J_s\left(exp\left({}^{e\,V}/_{KT}\right) - 1\right)$$

we obtain j(0.5V) = 1.5 × 10^{-13} A/cm^2 and j(−5V) = j$_s$.

6. The capacitance of a Au-n-GaAs Schottky diode is given by the relation 1/C^2 = 1.57 × 10^{15} − 2.12 × 10^{15} V, where C is expressed in F and V is in Volts. Taking the diode area to be 0.1 cm^2, calculate the barrier height and the dopant concentration.

Solution: (2.6)

7. From comparison of the de Broglie wavelength of electron with the depletion width of a contact metal-n-Si, estimate the electron concentration at which Schottky diode loses its rectifying characteristics. For the estimate, assume that the height of the potential barrier a the contact is half the value of the band gap at room temperature (E$_g$ = 1.12 eV), m*$_e$ = m$_0$, T = 300 K, and ε$_{Si}$ = 11.9.

Solution: (2.7)

φ_d = 0.74 V, n = 2.8 × 10^{17} cm^{-3}.

Part (17) Solved PROBLEMS

solar cell

Example 4.6

A 1 cm^2 silicon solar cell has a saturation current of 10^{-12} A and is illuminated with sunlight yielding a short-circuit photocurrent of 25 mA. Calculate the solar cell efficiency and fill factor.

Solution The maximum power is generated for:

$$\frac{d\,p}{d\,V_a} = 0 = I_s\left(e^{V_m/V_t} - 1\right) - I_{ph} + \frac{V_m}{V_T} I_s e^{V_m/V_t}$$

where the voltage, V$_m$, is the voltage corresponding to the maximum power point. This voltage is obtained by solving the following transcendental equation:

$$V_m = V_t\, ln\, \frac{1 + I_{ph}/I_s}{1 + V_m/V_t}$$

Using iteration and a starting value of 0.5 V one obtains the following successive values for V$_m$: V$_m$ = 0.5, 0.542, 0.540 V and the efficiency equals:

$$h = \left[\frac{V_m I_m}{P_{in}}\right] = \frac{0.54\ x\ 0.024}{0.1} = 13\%$$

The current, I$_m$, corresponding to the voltage, V$_m$, was calculated using equation (4.6.1) and the power of the sun was assumed 100 mW/cm^2. The fill factor equals:

$$full\ factor = \frac{V_m I_m}{V_{oc} I_{sc}} = \frac{0.54\ x\ 0.024}{0.62\ x\ 0.025} = 83\%$$

where the open circuit voltage is calculated using equation (4.6.1) and I = 0. The short circuit current equals the photocurrent